Published for SISSA by Springer

Received: *June 2, 2015*
Revised: *August 21, 2015*
Accepted: *September 23, 2015*
Published: *October 19, 2015*

JHEP10(2015)111

Three point functions in the large $\mathcal{N} = 4$ holography

Changhyun Ahn and Hyunsu Kim

Department of Physics, Kyungpook National University,
Taegu 41566, Korea

E-mail: ahn@knu.ac.kr, kimhyun@knu.ac.kr

Abstract: Sixteen higher spin currents with spins $(1, \frac{3}{2}, \frac{3}{2}, 2)$, $(\frac{3}{2}, 2, 2, \frac{5}{2})$, $(\frac{3}{2}, 2, 2, \frac{5}{2})$, and $(2, \frac{5}{2}, \frac{5}{2}, 3)$ were previously obtained in an extension of the large $\mathcal{N} = 4$ 'nonlinear' superconformal algebra in two dimensions. By carefully analyzing the zero-mode eigenvalue equations, three-point functions of bosonic (higher spin) currents are obtained with two scalars for any finite N (where $SU(N + 2)$ is the group of coset) and k (the level of spin-1 Kac Moody current). Furthermore, these 16 higher spin currents are implicitly obtained in an extension of large $\mathcal{N} = 4$ 'linear' superconformal algebra for generic N and k. The corresponding three-point functions are also determined. Under the large N 't Hooft limit, the two corresponding three-point functions in the nonlinear and linear versions coincide even though they are completely different for finite N and k.

Keywords: AdS-CFT Correspondence, Conformal and W Symmetry

ArXiv ePrint: 1506.00357

doi:10.1007/JHEP10(2015)111

Contents

1 Introduction

One consistency check in large $\mathcal{N}=4$ holography [1] can be described as the matching of correlation functions to show more dynamical information. The two dimensional conformal field theory (CFT) answer should be matched with the predictions from the bulk Vasiliev (or its generalization) theory. The simplest three-point functions involve two scalar primaries with one higher spin current. Then, the eigenvalue equations need to be obtained for the zero modes of the higher spin currents acting on the coset scalar primaries. The main motivation of [2] was to construct the higher spin currents for general N and k to explicitly see these eigenvalue equations. Here, the positive integer N appears in the group $G=\mathrm{SU}(N+2)$ of the $\mathcal{N}=4$ coset theory in two-dimensional CFT, while the positive integer k is the level of bosonic spin-1 (affine Kac-Moody) current. Relevant works are also discussed in [3, 4] for similar duality in the bosonic theory, and there exists a review paper [5] discussing relevant works in the context of higher spin anti-de Sitter (AdS)/CFT correspondence.

Before describing the eigenvalue equations, let us review the large $\mathcal{N}=4$ coset theory in two dimensions. Explicitly, the $\mathcal{N}=4$ coset theory is described by the following 'supersymmetric' coset:

$$\text{Wolf}\times\mathrm{SU}(2)\times\mathrm{U}(1)=\frac{\mathrm{SU}(N+2)}{\mathrm{SU}(N)},$$

where N is odd. The basic currents are given by the bosonic spin-1 current $V^a(z)$ and the fermionic spin-$\frac{1}{2}$ current $Q^b(z)$. The operator product expansion (OPE) between these currents does not have any singular term. The indices run over $a,b,\cdots=1,2,\cdots,\frac{(N+2)^2-1}{2}$, $1^*,2^*,\cdots,(\frac{(N+2)^2-1}{2})^*$. The number $(N+2)^2-1$ is the dimension of the $g=su(N+2)$ algebra. For extending the $\mathcal{N}=4$ 'nonlinear' superconformal algebra, the relevant coset is given by the Wolf space itself $\frac{\mathrm{SU}(N+2)}{\mathrm{SU}(N)\times\mathrm{SU}(2)\times\mathrm{U}(1)}$. For extending the $\mathcal{N}=4$ 'linear' superconformal algebra, the corresponding coset is given by the Wolf space multiplied by $\mathrm{SU}(2)\times\mathrm{U}(1)$, which is equivalent to the above coset $\frac{\mathrm{SU}(N+2)}{\mathrm{SU}(N)}$. In our previous work in [2], the explicit 16 lowest higher spin currents (which are multiple products of the above basic currents together with their derivatives) were expressed in terms of the Wolf space coset fields. These findings allow us to calculate the zero modes for the higher spin currents in terms of the generators of the $g=su(N+2)$ algebra because the zero modes of the spin-1 current satisfy the defining commutation relations of the underlying finite dimensional Lie algebra $su(N+2)$. Furthermore, all the OPEs between the higher spin currents and the spin-$\frac{1}{2}$ current are determined explicitly by construction.

The minimal representations of [1] are given by two representations. One minimal representation is given by $(0;f)$, where the nonnegative integer mode of the spin-1 current $V^a(z)$ in $\hat{su}(N+2)$ acting on the state $|(0;f)>$ vanishes. Under the decomposition of $su(N+2)$ into $su(N)\oplus su(2)$, the adjoint representation of $su(N+2)$ can be broken into the following representations: $(\mathbf{N+2})^{\mathbf{2}}-\mathbf{1}\rightarrow(\mathbf{N^2-1},\mathbf{1})\oplus(\mathbf{1},\mathbf{3})\oplus(\mathbf{1},\mathbf{1})\oplus(\mathbf{N},\mathbf{2})\oplus(\bar{\mathbf{N}},\mathbf{2})$. Among these representations, the fundamental representation for $su(N)$ is given by $(\mathbf{N},\mathbf{2})$.

Therefore, the representation $(0;f)$ corresponds to the representations $(\mathbf{N},\mathbf{2})$. Similarly, the representation $(0;\bar{f})$ corresponds to the representations $(\bar{\mathbf{N}},\mathbf{2})$. The corresponding states for the representation $(0;f)$ are given by the $-\frac{1}{2}$ mode of the spin-$\frac{1}{2}$ current $Q^a(z)$ acting on the vacuum $|0>$ (corresponding to (4.2) of [1]), where the index a is restricted to the $2N$ coset index.[1] As described in [1], from the OPEs between the spin-1 currents $A^{+i}(z)$ and the spin-$\frac{1}{2}$ current $Q^a(w)$ with coset index, the states $|(0;f)>$ are singlets with respect to the spin-1 currents $A^{+i}(z)$. The eigenvalue for the zero mode in the (higher spin) currents (multiple products of the above spin-1 and spin-$\frac{1}{2}$ currents) acting on this state can be obtained from the highest pole of the OPE between the (higher spin) current and the spin-$\frac{1}{2}$ current.[2]

The other minimal representation is given by $(f;0)$, where the positive half-integer mode of the spin-$\frac{1}{2}$ current $Q^a(z)$ in $\hat{su}(N+2)$ acting on the state $|(f;0)>$ vanishes. The positive integer modes of the spin-1 current should annihilate this state. They are singlets with respect to $su(N)$ in the $su(N+2)$ representation based on the fundamental representation. That is, the fundamental representation $(\mathbf{N}+\mathbf{2})$ of $su(N+2)$ transforms as a singlet $(\mathbf{1},\mathbf{2})_{-N}$ with respect to $su(N)$ under the branching $(\mathbf{N}+\mathbf{2}) \rightarrow (\mathbf{N},\mathbf{1})_2 \oplus (\mathbf{1},\mathbf{2})_{-N}$ with respect to $su(N) \oplus su(2) \oplus u(1)$. The indices 2 and $-N$ denote the U(1) charge, which will be described later in (3.10).[3] On the other hand, $(\mathbf{N},\mathbf{1})_2$ refers to the fundamental representation with respect to $su(N)$ and describes the light state $|(f;f)>$. For the state $|(f;0)>$, the $su(N+2)$ generator T_{a^*} corresponds to the zero mode of the spin-1 current $V^a(z)$ because the zero mode of the spin-1 current satisfies the commutation relation of the underlying finite-dimensional Lie algebra $su(N+2)$. Then, the nontrivial contributions to the zero-mode (of (higher spin) currents) eigenvalue equation associated with the state $|(f;0)>$ come from the multiple product of the spin-1 current $V^a(z)$ in the (higher spin) currents.[4] After substituting the $su(N+2)$ generator T_{a^*} into the zero mode of spin-1 current V_0^a in the multiple product of the (higher spin) currents, we obtain the $(N+2) \times (N+2)$ matrix acting on the state $|(f;0)>$. Then, the last 2×2 subdiagonal matrix is associated with the above $su(2) \oplus u(1)$ algebra. The eigenvalue can be obtained from each diagonal matrix element in this 2×2 matrix. Furthermore, the first $N \times N$

[1] We can further classify the two independent states denoted by $|(0;f)>_+$ with N coset indices and $|(0;f)>_-$ with other N coset indices [6] by emphasizing that $\pm$ refers to the doublet of $su(2)$.

[2] Furthermore, nontrivial states exist for the negative half-integer mode (as well as the $\frac{1}{2}$ mode) of the spin-$\frac{1}{2}$ current acting on the state $|(0;f)>$ because the action of the negative mode of the spin-$\frac{1}{2}$ current on the vacuum $|0>$ is nonzero. Note that the action of the $\frac{1}{2}$ mode for the spin-$\frac{1}{2}$ current on the state $|(0;f)>$ can be written in terms of the anticommutator of these modes acting on the vacuum $|0>$, which is nonzero [7–9]. The positive half-integer modes of the spin-$\frac{1}{2}$ current ($\frac{3}{2}, \frac{5}{2}, \cdots$ modes) acting on the state $|(0;f)>$ vanish.

[3] In this case, the states are further classified as $|(f;0)>_+$ and $|(f;0)>_-$ with explicit $su(2)$ indices. For the antifundamental representation of $su(N+2)$, the branching rule is $(\overline{\mathbf{N}+\mathbf{2}}) \rightarrow (\bar{\mathbf{N}},\mathbf{1})_{-2} \oplus (\mathbf{1},\mathbf{2})_N$ with respect to $su(N) \oplus su(2) \oplus u(1)$.

[4] When the spin-$\frac{1}{2}$ current is present in the (higher spin) current $\partial^i Q J^{(s-i-\frac{1}{2})}$, where $J^{(s-i-\frac{1}{2})}$ stands for composite field between the spin-1 currents with spin $(s-i-\frac{1}{2})$, then the zero mode contains $\sum_{p=-i+\frac{1}{2}}^{-\frac{1}{2}} J_{-p}^{(s-i-\frac{1}{2})}(\partial^i Q)_p|(f;0)>$. The group indices are ignored. From the mode expansion $Q^a(z) = \sum_{m=-\infty}^{\infty} \frac{Q^a_m}{z^{m+\frac{1}{2}}}$, the p mode of $(\partial^i Q)$ can be written as $(p+\frac{1}{2})(p+\frac{3}{2})\cdots(p+\frac{(2i-1)}{2})Q_p$ up to an overall factor. Therefore, for each p value in the summation, the coefficient in Q_p vanishes, and there is no contribution in the eigenvalue equation. Terms containing $(\partial^i Q)_p J_{-p}^{(s-i-\frac{1}{2})}|(f;0)>$ with negative p do not produce any contribution.

subdiagonal matrix provides the corresponding eigenvalues (for the higher spin currents) for the light state $|(f;f)>$, as mentioned before.

Under the large level limit, it has been conjectured that the perturbative Vasiliev theory is a subsector of the tensionless string theory [10]. The corresponding CFT is based on the small $\mathcal{N}=4$ linear superconformal algebra. For finite levels, the large $\mathcal{N}=4$ linear superconformal algebra plays an important role. Thus, it is natural to consider the extension of the large $\mathcal{N}=4$ linear superconformal algebra. The coset realization for the large $\mathcal{N}=4$ linear superconformal algebra has been performed by Saulina [11]. (See also [12].) Compared to the spin-$\frac{3}{2}$ currents in the large $\mathcal{N}=4$ nonlinear superconformal algebra, extra terms (which are cubic in the spin-$\frac{1}{2}$ currents) occur in the spin-$\frac{3}{2}$ currents of large $\mathcal{N}=4$ linear superconformal algebra. Moreover, the contractions between the basic currents in the coset $\frac{\mathrm{SU}(N+2)}{\mathrm{SU}(N)}$ contain extra coset indices corresponding to the above $\mathrm{SU}(2)\times\mathrm{U}(1)$. We can repeat the procedures described in [2] and would like to construct the higher spin currents in an extension of the large $\mathcal{N}=4$ linear superconformal algebra. In [13], the explicit coset realization for $N=3$ in the large $\mathcal{N}=4$ linear superconformal algebra has been found.

In section 2, the large $\mathcal{N}=4$ nonlinear superconformal algebra and its extension are reviewed [2, 13, 14], and the $su(N)$ subalgebra appears in the first $N\times N$ matrix inside of $(N+2)\times(N+2)$ matrix.

In section 3, based on section 2, the eigenvalue equations of the spin-2 stress-energy tensor are given for the above two minimal states. Next, the eigenvalue equations of higher spin currents with spins-$1,2$, and 3 for the above two minimal states are presented. The corresponding three-point functions are described.

In section 4, by summarizing the previous work of Saulina on the large $\mathcal{N}=4$ linear superconformal algebra, its extension is described.

In section 5, based on section 4, the eigenvalue equations of spin-2 stress-energy tensor (which is different from the one in section 2) are given for the above two minimal states. Next, the eigenvalue equations of higher spin currents with spins-$1,2$ and 3 for the above two minimal states are given. The corresponding three-point functions are described.

In section 6, the summary of this paper is given, and future directions are described briefly.

The Thielemans package [15] is used in this paper.[5]

[5] Another description by [16] exists (this is equivalent to the work of [1]) where the higher spin bosonic (and fermionic) currents are obtained from free bosons and free fermions. See also the work of [17]. For bosonic higher spin currents, the corresponding scalars, which appear in the three-point functions, are given by free fermions. As long as the three-point functions under the large N 't Hooft limit are concerned (therefore, they depend on the coupling λ in (3.1)), the analysis in [16] together with the constructions of the higher spin-s currents with $s=1,2,3$ should coincide with what we have found in this paper. See (3.27), for example. However, the higher order effects (or subleading orders) of $\frac{1}{c}$, where c is the central charge, are important in the study of marginal deformation in the Higgs phenomenon [18, 19], where the leading order of $\frac{1}{c}$ is taken because the quantum behavior occurs at finite N. As in the abstract, we will observe that the three-point functions in the nonlinear and linear versions behave differently for finite N. Note that for the large $\mathcal{N}=4$ linear superconformal algebra, the central charge c is given by $c=6(1-\lambda)(N+1)$. This aspect is one of the main motivations why we should consider the coset construction for the higher spin currents in a complicated way rather than the construction of free fermion theory, where the finite N behavior is not observed.

2 The large $\mathcal{N}=4$ nonlinear superconformal algebra and its extension in the Wolf space coset: review

To explicitly compare the previous results given in [1, 20] with our findings, we should associate the subgroup of the group in the coset theory with the first $N \times N$ matrix inside the $(N+2)\times(N+2)$ matrix. Then, the corresponding coset indices occur in the remaining row and column elements, except for the last 2×2 matrix elements.

2.1 The $\mathcal{N}=1$ Kac-Moody current algebra in component approach

Let us start by describing the OPEs of the $\hat{su}(N+2)$ current algebra. Here, T^a denotes the matrix representation of the Lie algebra $g = su(N+2)$ basis, satisfying the commutation relation $[T_a, T_b] = f_{ab}{}^c T_c$, where the indices run over $a, b, c, \cdots = 1, 2, \cdots, \frac{(N+2)^2-1}{2}, 1^*, 2^*, \cdots, (\frac{(N+2)^2-1}{2})^*$. The normalization is given as $g_{ab} = \frac{1}{2c_g} f_{ac}{}^d f_{bd}{}^c$, where c_g is the dual Coxeter number of the Lie algebra g. The operator product expansions between the spin-1 and the spin-$\frac{1}{2}$ currents are described as [21][6]

$$
\begin{aligned}
V^a(z)\, V^b(w) &= \frac{1}{(z-w)^2}\, k\, g^{ab} - \frac{1}{(z-w)}\, f^{ab}{}_c\, V^c(w) + \cdots , \\
Q^a(z)\, Q^b(w) &= -\frac{1}{(z-w)}\, (k+N+2)\, g^{ab} + \cdots , \qquad V^a(z)\, Q^b(w) = + \cdots .
\end{aligned} \tag{2.1}
$$

Here, k is a positive integer describing the level. Note that there is no singular term in the OPE between the spin-1 current $V^a(z)$ and the spin-$\frac{1}{2}$ current $Q^b(w)$.

2.2 The large $\mathcal{N}=4$ nonlinear superconformal algebra in the Wolf space coset

The Wolf space coset is given by

$$
\text{Wolf} = \frac{G}{H} = \frac{\text{SU}(N+2)}{\text{SU}(N)\times\text{SU}(2)\times\text{U}(1)}. \tag{2.2}
$$

The group indices are divided into

$$
\begin{aligned}
G \quad & \text{indices} : a, b, c, \cdots = 1, 2, \cdots, \frac{(N+2)^2-1}{2}, 1^*, 2^*, \cdots, \left(\frac{(N+2)^2-1}{2}\right)^*, \\
\frac{G}{H} \quad & \text{indices} : \bar{a}, \bar{b}, \bar{c}, \cdots = 1, 2, \cdots, 2N, 1^*, 2^*, \cdots, 2N^*.
\end{aligned} \tag{2.3}
$$

[6] In the work of [20], a different normalization for the spin-$\frac{1}{2}$ currents is used. The right hand side of their OPE between $\psi^{i,\alpha}(z)$ and $\bar{\psi}^{j,\beta}(w)$ has a first-order pole with weight 1. Our $\frac{1}{\sqrt{k+N+2}} Q^a(z)$ corresponds to their spin-$\frac{1}{2}$ currents.

For a given $(N+2)\times(N+2)$ matrix, we can associate the above $4N$ coset indices as follows [1]:

$$\left(\begin{array}{cccc|cc} & & & & * & * \\ & & & & * & * \\ & & & & \vdots & \vdots \\ & & & & * & * \\ & & & & * & * \\ \hline * & * & \cdots & * & * & \\ * & * & \cdots & * & * & \end{array}\right)_{(N+2)\times(N+2)}. \tag{2.4}$$

Because a different embedding is used in this study, different matrix representations of $h^i_{\bar{a}\bar{b}}$ and $d^0_{\bar{a}\bar{b}}$ will be given. However, the large $\mathcal{N}=4$ nonlinear superconformal algebra remains unchanged even though the 11 currents look different than expected.[7]

Then, the 11 currents of the large $\mathcal{N}=4$ nonlinear superconformal algebra in terms of $\mathcal{N}=1$ Kac-Moody currents $V^a(z)$ and $Q^{\bar{b}}(z)$ are obtained together with the three almost complex structures $h^i_{\bar{a}\bar{b}}(i=1,2,3)$. Furthermore, the three almost complex structures (h^1,h^2,h^3) are antisymmetric rank-two tensors and satisfy the algebra of imaginary quaternions [11]:

$$h^i_{\bar{a}\bar{c}}\, h^{j\bar{c}}{}_{\bar{b}} = \epsilon^{ijk}\, h^k_{\bar{a}\bar{b}} - \delta^{ij}\, g_{\bar{a}\bar{b}}. \tag{2.5}$$

By collecting the 11 currents in [2], the explicit 11 currents of the large $\mathcal{N}=4$ nonlinear superconformal algebra with (2.3) are given by[8]

$$\begin{aligned} G^0(z) &= \frac{i}{(k+N+2)}\, Q_{\bar{a}}\, V^{\bar{a}}(z), \qquad G^i(z) = \frac{i}{(k+N+2)}\, h^i_{\bar{a}\bar{b}}\, Q^{\bar{a}}\, V^{\bar{b}}(z), \\ A^{+i}(z) &= -\frac{1}{4N}\, f^{\bar{a}\bar{b}}{}_c\, h^i_{\bar{a}\bar{b}}\, V^c(z), \qquad A^{-i}(z) = -\frac{1}{4(k+N+2)}\, h^i_{\bar{a}\bar{b}}\, Q^{\bar{a}}\, Q^{\bar{b}}(z), \\ T(z) &= \frac{1}{2(k+N+2)^2}\left[(k+N+2)\, V_{\bar{a}}\, V^{\bar{a}} + k\, Q_{\bar{a}}\, \partial Q^{\bar{a}} + f_{\bar{a}\bar{b}c}\, Q^{\bar{a}}\, Q^{\bar{b}}\, V^c\right](z) \\ &\quad - \frac{1}{(k+N+2)} \sum_{i=1}^{3} \left(A^{+i}+A^{-i}\right)^2(z), \end{aligned} \tag{2.6}$$

[7]In the complex basis [22], the generators of $su(N+2)$ can be classified by a generator with starred indices and a generator with unstarred indices. The trace of any two generators with starred (unstarred) indices is equal to zero. Of course, the trace of two generators with mixed indices is not zero. This implies that there exists a vector space such that the scalar (or inner) product for each pair of vectors from this space is zero. Therefore, there are two isotropic subalgebras generated by the generators of $su(N+2)$ with starred indices and the generators of $su(N+2)$ with unstarred indices. See also the references [23, 24], where related subjects in the context of the Manin triple are discussed.

[8]The following relations occur between the spin-$\frac{3}{2}$ currents with double index notation and those with single index notation:

$$\begin{aligned} G_{11}(z) &= \frac{1}{\sqrt{2}}(G^1 - iG^2)(z), \qquad G_{12}(z) = -\frac{1}{\sqrt{2}}(G^3 - iG^0)(z), \\ G_{22}(z) &= \frac{1}{\sqrt{2}}(G^1 + iG^2)(z), \qquad G_{21}(z) = -\frac{1}{\sqrt{2}}(G^3 + iG^0)(z). \end{aligned}$$

where $i = 1, 2, 3$. The $G^\mu(z)$ currents are four supersymmetry currents, $A^{\pm i}(z)$ are six spin-1 generators of $\hat{su}(2)_k \oplus \hat{su}(2)_N$, and $T(z)$ is the spin-2 stress-energy tensor. In this paper, we will use these explicit results written in terms of the Wolf space coset fields.

Finally, the three almost complex structures (satisfying (2.5)) using $4N \times 4N$ matrices are given by

$$h^1_{\bar{a}\bar{b}} = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \quad h^2_{\bar{a}\bar{b}} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \qquad h^3_{\bar{a}\bar{b}} \equiv h^1_{\bar{a}\bar{c}}\, h^{2\bar{c}}{}_{\bar{b}}, \tag{2.7}$$

where each entry in (2.7) is an $N \times N$ matrix. Note that the corresponding relations in different embeddings appear in appendix B of [2].

2.3 Higher spin currents in the Wolf space coset (2.2)

The 16 lowest higher spin currents have the following four $\mathcal{N} = 2$ multiplets with spin contents

$$\begin{aligned} &\left(1, \frac{3}{2}, \frac{3}{2}, 2\right) : (T^{(1)}, T_+^{(\frac{3}{2})}, T_-^{(\frac{3}{2})}, T^{(2)}), && \left(\frac{3}{2}, 2, 2, \frac{5}{2}\right) : (U^{(\frac{3}{2})}, U_+^{(2)}, U_-^{(2)}, U^{(\frac{5}{2})}), \\ &\left(\frac{3}{2}, 2, 2, \frac{5}{2}\right) : (V^{(\frac{3}{2})}, V_+^{(2)}, V_-^{(2)}, V^{(\frac{5}{2})}), && \left(2, \frac{5}{2}, \frac{5}{2}, 3\right) : (W^{(2)}, W_+^{(\frac{5}{2})}, W_-^{(\frac{5}{2})}, W^{(3)}). \end{aligned} \tag{2.8}$$

The corresponding 16 higher spin currents will appear in section 4 in a different basis. The higher spin-1 current, which will be important in the linear version, also is[9]

$$T^{(1)}(z) = -\frac{1}{2(k+N+2)}\, d^0_{\bar{a}\bar{b}}\, f^{\bar{a}\bar{b}}{}_c V^c(z) + \frac{k}{2(k+N+2)^2}\, d^0_{\bar{a}\bar{b}}\, Q^{\bar{a}}\, Q^{\bar{b}}(z), \tag{2.9}$$

[9] There is also normalization with the overall sign with the following OPE:

$$T^{(1)}(z)T^{(1)}(w) = \frac{1}{(z-w)^2}\left[\frac{2Nk}{N+k+2}\right] + \cdots .$$

We can introduce the U(1) current $U(z)$ described in [20] from this higher spin-1 current. See also (4.20) and (4.38) of [20]. Then, we can easily see that their corresponding $U(z)$ is given by

$$\begin{aligned} U(z) &= (k+N+2)\left[-\frac{1}{2(k+N+2)}\, d^0_{\bar{a}\bar{b}}\, f^{\bar{a}\bar{b}}{}_c V^c(z)\right] - \frac{(N+2)(k+N+2)}{k}\left[\frac{k}{2(k+N+2)^2}\, d^0_{\bar{a}\bar{b}}\, Q^{\bar{a}}\, Q^{\bar{b}}(z)\right] \\ &= -\frac{1}{2}\, d^0_{\bar{a}\bar{b}}\, f^{\bar{a}\bar{b}}{}_c V^c(z) - \frac{(N+2)}{2(k+N+2)}\, d^0_{\bar{a}\bar{b}}\, Q^{\bar{a}}\, Q^{\bar{b}}(z). \end{aligned}$$

The OPEs between $U(z)$ and the coset components of the $\mathcal{N} = 1$ Kac-Moody currents $Q^a(w)$ and $V^b(w)$ satisfy the following OPEs:

$$\begin{aligned} U(z)\begin{pmatrix} Q^{\bar{A}} \\ Q^{\bar{A}^*} \end{pmatrix}(w) &= \mp\frac{1}{(z-w)}(N+2)\begin{pmatrix} Q^{\bar{A}} \\ Q^{\bar{A}^*} \end{pmatrix}(w) + \cdots , \\ U(z)\begin{pmatrix} V^{\bar{A}} \\ V^{\bar{A}^*} \end{pmatrix}(w) &= \mp\frac{1}{(z-w)}(N+2)\begin{pmatrix} V^{\bar{A}} \\ V^{\bar{A}^*} \end{pmatrix}(w) + \cdots . \end{aligned}$$

Note that the OPEs between $U(z)$ and the 11 currents in (2.6) are regular. The corresponding U(1) charges are given on the right-hand side. Then, the $T^{(1)}$ charges for the coset fields are given by $\pm\frac{k}{(k+N+2)}$ and $\mp\frac{(N+2)}{(k+N+2)}$.

where the rank-two tensor $d^0_{\bar{a}\bar{b}}$ is antisymmetric and satisfies the following properties:

$$d^0_{\bar{a}\bar{c}}\, d^{0\bar{c}}{}_{\bar{b}} = g_{\bar{a}\bar{b}}, \qquad d^0_{\bar{a}\bar{b}}\, f^{\bar{b}}{}_{\bar{c}d} = d^0_{\bar{c}\bar{b}}\, f^{\bar{b}}{}_{\bar{a}d}. \tag{2.10}$$

The tensorial structure in (2.9) is the same as the one in [2]. Furthermore, the $4N \times 4N$ matrix representation of $d^0_{\bar{a}\bar{b}}$ satisfying (2.10) is

$$d^0_{\bar{a}\bar{b}} = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \tag{2.11}$$

Each entry is an $N \times N$ matrix as before. The corresponding d tensor in [2] appears in appendix B of [2].

Let us define the four higher spin-$\frac{3}{2}$ currents $G'^{\mu}(z)$ from the first-order pole of the following OPE:

$$G^{\mu}(z)\, T^{(1)}(w) = \frac{1}{(z-w)}\, G'^{\mu}(w) + \cdots . \tag{2.12}$$

Then, the first-order pole in (2.12) provides

$$G'^{\mu}(z) = \frac{i}{(k+N+2)}\, d^{\mu}_{\bar{a}\bar{b}}\, Q^{\bar{a}}\, V^{\bar{b}}(z), \tag{2.13}$$

where $d^{\mu}_{\bar{a}\bar{b}} \equiv d^{0\bar{c}}_{\bar{a}}\, h^{\mu}_{\bar{c}\bar{b}}$ and $h^0_{\bar{a}\bar{b}} \equiv g_{\bar{a}\bar{b}}$ with (2.11). These four independent higher spin-$\frac{3}{2}$ currents also appear in the linear version in section 4.

Then, the remaining 15 currents of (2.8) can be written in terms of the $\mathcal{N}=1$ Kac-Moody currents $V^a(z), Q^{\bar{b}}(z)$, the three almost complex structures $h^i_{\bar{a}\bar{b}}$, the antisymmetric rank-two tensor $d^0_{\bar{a}\bar{b}}$, and the symmetric rank-two tensors $d^i_{\bar{a}\bar{b}} (\equiv d^{0\bar{c}}_{\bar{a}}\, h^i_{\bar{c}\bar{b}})$ as in [2].

3 Three-point functions in an extension of large $\mathcal{N}=4$ nonlinear superconformal algebra

This section describes the three-point functions with scalars for the currents of spins $s = 1, 2$ and the higher spin currents of spins $s = 1, 2, 3$ explained in previous section. The large N limit is defined by [1]

$$N, k \to \infty, \qquad \lambda \equiv \frac{N+1}{N+k+2} \quad \text{fixed}. \tag{3.1}$$

As described in the introduction, we primarily focus on the two simplest states $|(f;0)>$ and $|(0;f)>$ given in [1].

3.1 Eigenvalue equations for the spin-2 current in the $\mathcal{N}=4$ nonlinear superconformal algebra

Let us focus on the eigenvalue equations for the stress-energy tensor (2.6) acting on the above two states.

3.1.1 Eigenvalue equation for the spin-2 current acting on the state $|(f;0)>$

As described in the introduction, the terms containing the fermionic spin-$\frac{1}{2}$ currents $Q^a(z)$ do not contribute to the eigenvalue equation when we calculate the zero-mode eigenvalues for the bosonic spin-s current $J^{(s)}(z)$ acting on state $|(f;0)>$. The zero mode of the spin-1 current satisfies the commutation relation of the underlying finite-dimensional Lie algebra $su(N+2)$. For the state $|(f;0)>$, the generator T_{a^*} corresponds to the zero mode V_0^a as follows (see also [25]):

$$V_0^a|(f;0)>= T_{a^*}|(f;0)> . \tag{3.2}$$

Then, the eigenvalues are encoded in the last 2×2 diagonal matrix. Note that the nonvanishing components of the metric are given by $g_{aa^*}=1$.

For example, we can calculate the conformal dimension of $|(f;0)>$ when $N=3$. The explicit form for the stress-energy tensor is given by (2.6). The only $Q^a(z)$ independent terms are given by the first term and the $A^{+i}A^{+i}(z)$-dependent term because the other terms contain $Q^a(z)$ explicitly. Then, the eigenvalue equation for the zero mode of the spin-2 current acting on the state $|(f;0)>$ leads to

$$\begin{aligned}
T_0|(f;0)> &\sim \left[\frac{1}{2(k+5)}V_{\bar{a}}V^{\bar{a}}-\frac{1}{(k+5)}\sum_{i=1}^{3}A^{+i}A^{+i}\right]_0 |(f;0)> \\
&= \left[\frac{1}{2(k+5)}\left(\sum_{a=1}^{6}V^aV^{a^*}+\sum_{a=1}^{6}V^{a^*}V^a\right)\right]_0|(f;0)>+\frac{1}{(k+5)}\left[-\sum_{i=1}^{3}A^{+i}A^{+i}\right]_0|(f;0)> \\
&= \left[\frac{1}{2(k+5)}\left(\sum_{a=1}^{6}T_{a^*}T_a+\sum_{a=1}^{6}T_aT_{a^*}\right)\right]|(f;0)>+\frac{1}{(k+5)}l^+(l^++1)|(f;0)> \\
&= \frac{3}{2(k+5)}|(f;0)>+\frac{1}{(k+5)}\frac{3}{4}|(f;0)>=\frac{9}{4(k+5)}|(f;0)>,
\end{aligned} \tag{3.3}$$

where $\sim$ in the first line of (3.3) means that we ignore the terms including $Q^a(z)$. In the second line, the summation over the coset indices $\bar{a}=1,2,\cdots,6,1^*,2^*,\cdots,6^*$ is taken explicitly. In the third line, the corresponding $su(5)$ generators using the condition (3.2) are replaced. Moreover, the eigenvalue equation for the zero mode of the quadratic spin-1 currents is used, where l^+ is the spin of the $\hat{su}(2)$ algebra. In the fourth line, the $su(5)$ matrix product is done, and we take 3 from the last 2×2 diagonal matrix.[10]

[10]The highest weight states of the large $\mathcal{N}=4$ (non)linear superconformal algebra can be characterized by the conformal dimension h and two (iso)spins $l^\pm$ of $\hat{su}(2)\oplus\hat{su}(2)$ [6]:

$$\left[-\sum_{i=1}^{3}A^{+i}A^{+i}\right]_0|\mathrm{hws}>=l^+(l^++1)\,|\mathrm{hws}>,\qquad \left[-\sum_{i=1}^{3}A^{-i}A^{-i}\right]_0|\mathrm{hws}>=l^-(l^-+1)\,|\mathrm{hws}>. \tag{3.4}$$

For example, in $g=su(5)$, the expressions (2.6) imply that

$$\left[-\sum_{i=1}^{3}A^{+i}A^{+i}\right]_0|(f;\star)>=\left(\begin{array}{ccc|cc} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ \hline 0&0&0&\frac{3}{4}&0\\ 0&0&0&0&\frac{3}{4}\end{array}\right)|(f;\star)>,\qquad \left[-\sum_{i=1}^{3}A^{-i}A^{-i}\right](z)\,Q^{\bar{A}^*}(w)|_{\frac{1}{(z-w)^2}}=\frac{3}{4}Q^{\bar{A}^*}(w),$$

where the representation $\star=0$ (trivial representation) or f (fundamental representation) of $su(3)$. Note that, although all of these expressions are written in terms of fields without boldface (the nonlinear version), Now, we want to obtain the general N-dependence for the above eigenvalue equation. From the similar calculations for $N = 5, 7, 9$ where all the (higher spin) currents are known explicitly, we can find the N-dependence of the corresponding $(N + 2) \times (N + 2)$ matrix. In the generalization of third line of (3.3), the first $2N$ terms contribute to the first N diagonal elements, 2, and the last $2N$ terms contribute to the last two diagonal elements, N. That is, the first N diagonal elements are given by 2, and the remaining last two diagonal elements are given by N. The eigenvalues $\frac{3}{4}$ obtained in the footnote 10 hold for any N. The total contribution is given by $(\frac{N}{2} + \frac{3}{4})$ multiplied by an obvious overall factor $\frac{1}{(k+N+2)}$. Therefore, the eigenvalue equation (for generic N) for the zero mode of the spin-2 current, which provides the conformal dimension for the state $|(f; 0) >$, is given by

$$T_0|(f;0) >= \left[\frac{(2N+3)}{4(k+N+2)}\right] |(f;0) >, \tag{3.5}$$

where the eigenvalue is the same value as $h(f;0)$ given in [1]. We confirm (3.5) for $N = 3, 5, 7, 9$. We can also check that this leads to the following reduced eigenvalue equation: $T_0|(f;0) >= \frac{\lambda}{2}|(f;0) >$ under the large N 't Hooft limit (3.1).

3.1.2 Eigenvalue equation for the spin-2 current acting on state $|(0; f) >$

Let us consider the next simplest representation. When we calculate the eigenvalue equations for state $|(0; f) >$, because the state is given by [1, 6]

$$|(0;f) >= \frac{1}{\sqrt{k+N+2}}\, Q^{\bar{A}^*}_{-\frac{1}{2}}|0 >, \qquad \bar{A}^* = 1^*, 2^*, \cdots, 2N^*, \tag{3.6}$$

the OPEs between the (higher spin) currents $J^{(s)}(z)$ and $Q^{\bar{A}^*}(w)$ are needed. Here, the notation for the index is rather confusing. The index in $\bar{A}^*$ contains the bar, while the numerical components do not contain the bar. In (2.3), there is no bar in the numerical components. We need only the coefficient of the highest-order pole $\frac{1}{(z-w)^s}$ in the OPEs. The lower singular terms do not contribute to the zero-mode eigenvalue equations. Let us denote the highest-order pole as follows [17, 27]:

$$J^{(s)}(z)\, Q^{\bar{A}^*}(w)|_{\frac{1}{(z-w)^s}} = j(s)\, Q^{\bar{A}^*}(w), \tag{3.7}$$

where $j(s)$ stands for the corresponding coefficient of highest order pole. We can write $J_0^{(s)}|(0; f) >$ as $\frac{1}{\sqrt{k+N+2}}[J_0^{(s)}, Q^{\bar{A}^*}_{-\frac{1}{2}}]|0 >$, and this commutator acting on the vacuum can be written in terms of $\frac{1}{\sqrt{k+N+2}} j(s)\, Q^{\bar{A}^*}_{-\frac{1}{2}}|0 >$. Then, we obtain the following eigenvalue equation for the zero mode of the spin-s current together with (3.6) and (3.7)

$$J_0^{(s)}|(0;f) >= j(s)|(0;f) >, \tag{3.8}$$

it is also true that all of these can be replaced by fields with boldface (the linear version). We can see $l^+(f;0) = \frac{1}{2}$ (from the two eigenvalues $\frac{3}{4}$), $l^+(f;f) = 0$ (from the three eigenvalues 0), and $l^-(0;f) = \frac{1}{2}$ (from the coefficient of the second-order pole $\frac{3}{4}$). Then, the state $|(f;0) >$ has $l^+ = \frac{1}{2}$, $l^- = 0$, the state $|(0;f) >$ has $l^+ = 0$, $l^- = \frac{1}{2}$, and the state $|(f;f) >$ has $l^\pm = 0$. The eigenvalues for l^- will be explained in next subsection. Note the (-1) sign in the left-hand side of (3.4) comes from the anti-Hermitian property [6, 26].

where the explicit relation between the current and its mode is given by $J^{(s)}(z) = \sum_{n=-\infty}^{\infty} \frac{J_n^{(s)}}{z^{n+s}}$. Therefore, to determine the above eigenvalue $j(s)$, we should calculate the explicit OPEs between the corresponding (higher spin) currents and the spin-$\frac{1}{2}$ current and select the highest-order pole.

Let us consider the eigenvalue equation for the spin-2 current acting on the above state. Because the OPE between the spin-1 current $A^{+i}(z)$ and the spin-$\frac{1}{2}$ current $Q^{\bar{A}^*}(w)$ is regular, the terms containing $A^{+i}(z)$ in $T(z)$ of (2.6) do not contribute to the highest-order pole. Furthermore, the terms containing the spin-1 current $V^a(z)$ do not contribute to the highest-order pole. Therefore, the relevant terms in $T(z)$ are given purely by the spin-$\frac{1}{2}$ current-dependent terms (by completely ignoring the spin-1 current-dependent terms). Then, the conformal dimension of $|(0;f)>$ is

$$\begin{aligned} T_0|(0;f)> &\sim \left[\frac{k}{2(k+N+2)^2} Q_{\bar{a}} \partial Q^{\bar{a}} - \frac{1}{(k+N+2)} \sum_{i=1}^{3} A^{-i} A^{-i}\right]_0 |(0;f)> \\ &= \left[\frac{k}{2(k+N+2)^2} Q_{\bar{a}} \partial Q^{\bar{a}}\right]_0 |(0;f)> + \frac{1}{(k+N+2)} \left[-\sum_{i=1}^{3} A^{-i} A^{-i}\right]_0 |(0;f)> \\ &= \frac{k}{2(k+N+2)} |(0;f)> + \frac{1}{(k+N+2)} l^-(l^-+1)|(0;f)> \\ &= \left[\frac{(2k+3)}{4(N+k+2)}\right] |(0;f)> . \end{aligned} \tag{3.9}$$

In the first line of (3.9), the spin-1 current-dependent terms are ignored. In the third line, we have used the fact that the eigenvalue equation $[Q_{\bar{a}} \partial Q^{\bar{a}}]_0 |(0;f)> = (k+N+2)|(0;f)>$ (see (3.8)) can be obtained because the highest-order pole gives the corresponding eigenvalue $Q_{\bar{a}} \partial Q^{\bar{a}}(z)\, Q^{\bar{A}^*}(w)|_{\frac{1}{(z-w)^2}} = (k+N+2) Q^{\bar{A}^*}(w)$ (see (3.7)), which can be confirmed from the defining relation in (2.1). Furthermore, the characteristic eigenvalue equation for the $\hat{su}(2)$ algebra described in footnote 10 is used. The total contribution is given by $(\frac{k}{2} + \frac{3}{4})$ multiplied by the overall factor $\frac{1}{(N+k+2)}$. The above eigenvalue is exactly the same as that of $h(0;f)$ described in [1]. Under the large N 't Hooft limit (3.1), the eigenvalue equation implies that $T_0|(0;f)> = \frac{1}{2}(1-\lambda)|(0;f)>$.

There exists $N \leftrightarrow k$ and $0 \leftrightarrow f$ symmetry between the above two eigenvalue equations (3.5) and (3.9). In the classical symmetry (in the large N 't Hooft limit), this is equivalent to the exchange of $\lambda \leftrightarrow (1-\lambda)$ and $0 \leftrightarrow f$. The U-charge corresponding to twice of $\hat{u}$ charge in [1] can be determined as follows:

$$U_0|(f;0)> = -N|(f;0)>, \qquad U_0|(0;f)> = (N+2)|(0;f)>, \tag{3.10}$$

where we use the same normalization as in [20]. The second relation can be seen from the explicit OPE result in footnote 9, where the eigenvalue $(N+2)$ appears in the OPE between $U(z)$ and $Q^{\bar{A}*}(w)$. The detailed description for these eigenvalue equations will be given in next subsection.[11]

[11]Explicitly, for $N = 3$, we have $U(z) = \frac{1}{4}(3\sqrt{10} + 5i\sqrt{6})V^{12}(z) + \frac{1}{4}(3\sqrt{10} - 5i\sqrt{6})V^{12^*}(z) +$

3.2 Eigenvalue equation for the higher spin currents of spins $1, 2$ and 3

Now, we can consider the eigenvalue equations for the higher spin currents by following the descriptions in previous subsection.

3.2.1 Eigenvalue equation for the higher spin-1 current acting on the states $|(f;0)>$ and $|(0;f)>$

From the explicit expression in (2.9), we begin with the first term, which does not contain the spin-$\frac{1}{2}$ current, applied to the condition (3.2), and determine the last two diagonal matrix elements for $N=3$. We find

$$\begin{aligned} T_0^{(1)}|(f;0)> &\sim \left[-\frac{1}{2(k+5)}\, d^0_{\bar{a}\bar{b}}\, f^{\bar{a}\bar{b}}{}_c V^c\right]_0 |(f;0)> \\ &= \left[-\frac{1}{2(k+5)}\left(-\frac{3\sqrt{10}+5i\sqrt{6}}{2}V^{12}-\frac{3\sqrt{10}-5i\sqrt{6}}{2}V^{12^*}\right)\right]_0 |(f;0)> \\ &= \left[-\frac{1}{2(k+5)}\left(-\frac{3\sqrt{10}+5i\sqrt{6}}{2}T_{12^*}-\frac{3\sqrt{10}-5i\sqrt{6}}{2}T_{12}\right)\right] |(f;0)> \\ &= -\frac{3}{(k+5)}|(f;0)>\,. \end{aligned} \tag{3.11}$$

In the first line of (3.11), we ignore the spin-$\frac{1}{2}$-dependent part. In the second line, we can substitute the d tensor and f structure constant for $N=3$ from section 2. From the third line to the fourth line, the explicit generators are substituted, and eigenvalues 6 from the last 2×2 diagonal matrix are taken. We want to obtain the general N behavior for the above eigenvalue equation. We can find the N-dependence of the above 5×5 matrix using a similar calculation for the next several values for N:$N=5,7,9$. The $(N+2)\times(N+2)$ matrix in $g=su(N+2)$ is given by $\mathrm{diag}(-4,\cdots,-4,2N,2N)$ with the obvious overall factor $-\frac{1}{2(N+k+2)}$. The N-dependence appears in the last 2×2 diagonal matrix. Therefore,

---

$\frac{5}{(5+k)}\sum_{a=1}^{6} Q^a Q^{a^*}(z)$. As shown in (3.3), we can construct the following eigenvalue equation for $g=su(5)$:

$$U_0|(f;\star)>= \left(\begin{array}{ccc|cc} 2&0&0&0&0\\ 0&2&0&0&0\\ 0&0&2&0&0\\ \hline 0&0&0&-3&0\\ 0&0&0&0&-3 \end{array}\right)|(f;\star)>,$$

where the representation $\star$ is the trivial representation 0 or the fundamental representation f in $su(3)$. The eigenvalues -3 living in the lower 2×2 diagonal matrix can be generalized to $-N$ according to the next subsection. By reading the first three eigenvalues 2, which are valid for generic N, the U-charge for the state $|(f;f)>$ is given by 2 [1]. Furthermore, we can determine the conformal dimension for the 'light' state $|(f;f)>$ using (3.3) and its generalization. Because $l^+(f;f)=0$ (from the analysis in the footnote 10), the eigenvalue equation is $T_0|(f;f)>=\frac{1}{2(k+N+2)}\times 2|(f;f)>=\frac{1}{(k+N+2)}|(f;f)>$ as observed in [1]. Under the large N 't Hooft limit, we have $T_0|(f;f)>=\frac{\lambda}{(N+1)}|(f;f)>$, and this vanishes.

the eigenvalue equation for generic N can be summarized by[12]

$$T_0^{(1)}|(f;0)>= -\left[\frac{N}{(k+N+2)}\right]|(f;0)> . \tag{3.12}$$

The zero-mode eigenvalue equation for the state $|(0;f)>$ can be obtained from the explicit OPE between the second term in (2.9) and the spin-$\frac{1}{2}$ current $Q^{\bar{A}^*}(w)$, and we obtain

$$T_0^{(1)}|(0;f)>= -\left[\frac{k}{(k+N+2)}\right]|(0;f)> . \tag{3.13}$$

An $N \leftrightarrow k$ and $0 \leftrightarrow f$ symmetry obviously exists between the two eigenvalue equations in (3.12) and (3.13).[13] Under the large N 't Hooft limit (3.1), we obtain the following eigenvalue equations:

$$T_0^{(1)}|(f;0)>= -\lambda|(f;0)>, \qquad T_0^{(1)}|(0;f)>= -(1-\lambda)|(0;f)> . \tag{3.14}$$

Compared to the previous U charge in (3.10), the above higher spin-1 current preserves the $N \leftrightarrow k$ symmetry. If we replace the fundamental representation f with the antifundamental representation $\bar{f}$ in (3.12) and (3.13), additional minus signs appear on the right-hand side. Note that the corresponding equations are given by $V_0^a|(\bar{f};0)>= -(T_{a^*})^T|(\bar{f};0)>$ and $|(0;\bar{f})>= \frac{1}{\sqrt{k+N+2}}Q^{\bar{A}}_{-\frac{1}{2}}|0>$ associated with (3.2) and (3.6).

3.2.2 Eigenvalue equation for the higher spin-2 currents acting on the states $|(f;0)>$ and $|(0;f)>$

To understand the eigenvalue equations for the higher spin-2 currents, we should classify the $|(f;0)>$ states into the following two types of column vectors:

$$|(f;0)>_+ = (0,\cdots,0,1,0)^T, \qquad |(f;0)>_- = (0,\cdots,0,0,1)^T. \tag{3.15}$$

These vectors are **2** under the $su(2)$ and transform as singlets under $su(N)$ characterized by the first N zeros in (3.15). Moreover, these vectors have nontrivial U(1) charges (3.10).

On the other hand, the $|(0;f)>$ states are expressed by the following forms:

$$|(0;f)>_+ : \frac{1}{\sqrt{k+N+2}}Q^{1^*}_{-\frac{1}{2}}|0>, \qquad \cdots, \qquad \frac{1}{\sqrt{k+N+2}}Q^{N^*}_{-\frac{1}{2}}|0>,$$

[12] We can determine the eigenvalue equation for the 'light' state $|(f;f)>$ by taking the eigenvalues of -4 living in the first $N\times N$ diagonal matrix as follows:

$$T_0^{(1)}|(f;f)>= -\frac{1}{2(k+N+2)}\times(-4)|(f;f)>= \left[\frac{2}{(k+N+2)}\right]|(f;f)> .$$

[13] We can find the above two eigenvalue equations by indirectly using the U-charge introduced in (3.10) because the first term (second term) of $T^{(1)}(z)$ in (2.9) contributes to the zero-mode eigenvalue equation for the state $|(f;0)>$ ($|(0;f)>$). See footnote 9. By correctly obtaining the numerical factors, we find that one of these factors is given by $T_0^{(1)}|(f;0)>= \frac{1}{(k+N+2)}U_0|(f;0)>= -\frac{N}{(k+N+2)}|(f;0)>$, where the spin-$\frac{1}{2}$-dependent term is ignored. The other factor is given by $T_0^{(1)}|(0;f)>= -\frac{k}{(k+N+2)(N+2)}U_0|(0;f)>= -\frac{k}{(k+N+2)}|(0;f)>$, where the spin-1-dependent term is ignored.

$$|(0;f)>_- \; : \; \frac{1}{\sqrt{k+N+2}} Q^{(N+1)^*}_{-\frac{1}{2}} |0>, \qquad \cdots, \qquad \frac{1}{\sqrt{k+N+2}} Q^{(2N)^*}_{-\frac{1}{2}} |0> . \quad (3.16)$$

These states are the fundamental representation $\mathbf{N}$ under $su(N)$ and transform as a doublet under $su(2)$.[14]

The eigenvalue equations for the higher spin-2 (primary) current $T^{(2)}(z)$ acting on (3.16) and (3.15) are summarized by

$$\begin{aligned}
T_0^{(2)}|(f;0)>_+ &= \left[\frac{N(2Nk+2N-k)}{2(N+k+2)(2Nk+N+k)}\right]|(f;0)>_+, \\
T_0^{(2)}|(f;0)>_- &= -\left[\frac{Nk(2N+3)}{2(N+k+2)(2Nk+N+k)}\right]|(f;0)>_-, \\
T_0^{(2)}|(0;f)>_+ &= \left[\frac{k(2kN+2k-N)}{2(N+k+2)(2Nk+N+k)}\right]|(0;f)>_+, \\
T_0^{(2)}|(0;f)>_- &= -\left[\frac{Nk(2k+3)}{2(N+k+2)(2Nk+N+k)}\right]|(0;f)>_- . \qquad (3.17)
\end{aligned}$$

These are the first examples where all of the eigenvalues are different.[15] There exists $N \leftrightarrow k$ and $0 \leftrightarrow f$ symmetry in (3.17) such that

$$\left[T_0^{(2)}|(f;0)>_\pm\right]_{N\leftrightarrow k, 0\leftrightarrow f} = T_0^{(2)}|(0;f)>_\pm . \qquad (3.18)$$

The states $|(f;0)>_\pm$ are changed into $|(0;f)>_\pm$ and vice versa. Furthermore, if we replace the fundamental representation f with the antifundamental representation $\bar{f}$ in (3.17), then the right-hand sides remain unchanged.

The higher spin-2 primary current $\tilde{W}^{(2)}(z)$ was defined in [2] as follows:

$$\tilde{W}^{(2)}(z) \equiv \left[W^{(2)} - \frac{2kN}{(k+N+2kN)} T\right](z). \qquad (3.19)$$

Then, the corresponding eigenvalue equations from (3.15), (3.16) and (3.19) can be obtained as follows:

$$\begin{aligned}
\tilde{W}_0^{(2)}|(f;0)>_+ &= -\left[\frac{Nk(2N+3)}{2(N+k+2)(2Nk+N+k)}\right]|(f;0)>_+, \\
\tilde{W}_0^{(2)}|(f;0)>_- &= \left[\frac{N(2Nk+2N-k)}{2(N+k+2)(2Nk+N+k)}\right]|(f;0)>_-,
\end{aligned}$$

[14] Let us comment on the nontrivial action of the spin-1 currents into the above two states. The results are given as follows:

$$A_0^{+\pm}|(f;0)>_\mp = -i|(f;0)>_\pm, \qquad A_0^{-\pm}|(0;f)>_\mp = i|(0;f)>_\pm .$$

We have nontrivial eigenvalues as these apply to one more zero mode at each expression. The corresponding three-point functions can be described.

[15] More precisely, the previous eigenvalue equations (3.5), (3.9), (3.12), and (3.13) can be rewritten as $T_0|(f;0)>_\pm = \frac{(2N+3)}{4(N+k+2)}|(f;0)>_\pm$, $T_0|(0;f)>_\pm = \frac{(2k+3)}{4(N+k+2)}|(0;f)>_\pm$, $T_0^{(1)}|(f;0)>_\pm = -\frac{N}{(N+k+2)}|(f;0)>_\pm$, and $T_0^{(1)}|(0;f)>_\pm = -\frac{k}{(N+k+2)}|(0;f)>_\pm$, respectively.

$$\begin{aligned}
\tilde{W}_0^{(2)}|(0;f)>_+ &= \left[\frac{k(2Nk+2k-N)}{2(N+k+2)(2Nk+N+k)}\right]|(0;f)>_+,\\
\tilde{W}_0^{(2)}|(0;f)>_- &= -\left[\frac{Nk(2k+3)}{2(N+k+2)(2Nk+N+k)}\right]|(0;f)>_- .
\end{aligned} \tag{3.20}$$

There exists an $N \leftrightarrow k$ symmetry in (3.20) such that

$$\left[\tilde{W}_0^{(2)}|(f;0)>_\pm\right]_{N\leftrightarrow k, 0\leftrightarrow f, +\leftrightarrow -} = \tilde{W}_0^{(2)}|(0;f)>_\mp . \tag{3.21}$$

Note that the states $|(f;0)>_\pm$ are changed into $|(0;f)>_\mp$ and vice versa. Furthermore, if we replace the fundamental representation f with the antifundamental representation $\bar{f}$ in (3.20), the right-hand sides remain unchanged.

We can easily see that precise relations exist between the above eigenvalue equations:

$$\left[T_0^{(2)}|(f;0)>_\pm\right]_{+\leftrightarrow -} = \tilde{W}_0^{(2)}|(f;0)>_\mp, \qquad T_0^{(2)}|(0;f)>_\pm = \tilde{W}_0^{(2)}|(0;f)>_\pm . \tag{3.22}$$

Furthermore, under the large N 't Hooft limit (3.1), the above eigenvalue equations (3.17) and (3.20) (or (3.22)) become

$$\begin{aligned}
T_0^{(2)}|(f;0)>_\pm &= \pm\frac{1}{2}\lambda|(f;0)>_\pm, \qquad T_0^{(2)}|(0;f)>_\pm = \pm\frac{1}{2}(1-\lambda)|(0;f)>_\pm,\\
\tilde{W}_0^{(2)}|(f;0)>_\pm &= \mp\frac{1}{2}\lambda|(f;0)>_\pm, \qquad \tilde{W}_0^{(2)}|(0;f)>_\pm = \pm\frac{1}{2}(1-\lambda)|(0;f)>_\pm .
\end{aligned} \tag{3.23}$$

Up to the overall sign, these relations (3.23) behave similarly to those in the spin-2 current described in subsection 3.1.

For the other spin-2 currents, we obtain the following nonzero results[16]

$$\begin{aligned}
\left[U_+^{(2)}\right]_0|(0;f)>_+ &= \left[\frac{k}{(N+k+2)}\right]|(0;f)>_- \rightarrow (1-\lambda)|(0;f)>_-,\\
\left[U_-^{(2)}\right]_0|(f;0)>_+ &= -\left[\frac{N}{(N+k+2)}\right]|(f;0)>_- \rightarrow -\lambda|(f;0)>_-,\\
\left[V_+^{(2)}\right]_0|(f;0)>_- &= \left[\frac{N}{(N+k+2)}\right]|(f;0)>_+ \rightarrow \lambda|(f;0)>_+,\\
\left[V_-^{(2)}\right]_0|(0;f)>_- &= -\left[\frac{k}{(N+k+2)}\right]|(0;f)>_+ \rightarrow -(1-\lambda)|(0;f)>_+ .
\end{aligned}$$

---

[16] More precisely, we have

$$\begin{aligned}
\left[U_+^{(2)}\right]_0 \frac{1}{\sqrt{N+k+2}}Q^{a^*}_{-\frac{1}{2}}|0> &= \left[\frac{k}{(N+k+2)}\right]\frac{1}{\sqrt{N+k+2}}Q^{(a+N)^*}_{-\frac{1}{2}}|0>,\\
\left[V_-^{(2)}\right]_0 \frac{1}{\sqrt{N+k+2}}Q^{(a+N)^*}_{-\frac{1}{2}}|0> &= -\left[\frac{k}{(N+k+2)}\right]\frac{1}{\sqrt{N+k+2}}Q^{a^*}_{-\frac{1}{2}}|0>,
\end{aligned}$$

where $a = 1, 2, \cdots, N$.

JHEP10(2015)111

3.2.3 Eigenvalue equation for the higher spin-3 current acting on states $|(f;0)>$ and $|(0;f)>$

The eigenvalue equations of the zero mode of the higher spin-3 current $W^{(3)}(z)$ are described as[17]

$$
\begin{aligned}
W_0^{(3)}|(f;0)>_+ &= \left[\frac{N(2N+k+1)(12Nk+10k+4N-1)}{3(N+k+2)^2(6Nk+5N+5k+4)}\right]|(f;0)>_+,\\
W_0^{(3)}|(f;0)>_- &= \left[\frac{N(24N^2k+12Nk^2+8N^2+10k^2+48Nk-6N+43k+1)}{3(N+k+2)^2(6Nk+5N+5k+4)}\right]|(f;0)>_-,\\
W_0^{(3)}|(0;f)>_+ &= -\left[\frac{k(24k^2N+12kN^2+8k^2+10N^2+48kN-6k+43N+1)}{3(N+k+2)^2(6Nk+5N+5k+4)}\right]|(0;f)>_+,\\
W_0^{(3)}|(0;f)>_- &= -\left[\frac{k(2k+N+1)(12Nk+10N+4k-1)}{3(N+k+2)^2(6Nk+5N+5k+4)}\right]|(0;f)>_- \,.
\end{aligned}
\tag{3.24}
$$

There exists an $N \leftrightarrow k$ symmetry in (3.24) such that

$$
\left[W_0^{(3)}|(f;0)>_\pm\right]_{N\leftrightarrow k,0\leftrightarrow f,+\leftrightarrow -} = -W_0^{(3)}|(0;f)>_\mp, \tag{3.25}
$$

which looks similar to (3.21) up to the sign. Furthermore, if we replace the fundamental representation f with the antifundamental representation $\bar{f}$ in (3.24), an extra minus sign appears in the right-hand side.

Under the large N 't Hooft limit (3.1), we have

$$
W_0^{(3)}|(f;0)> = \frac{2}{3}\lambda(1+\lambda)|(f;0)>, W_0^{(3)}|(0;f)> = -\frac{2}{3}(1-\lambda)(2-\lambda)|(0;f)> . \tag{3.26}
$$

The following relation holds $\left[W_0^{(3)}|(f;0)>\right]_{\lambda\to(1-\lambda),0\leftrightarrow f} = -W_0^{(3)}|(0;f)>$. This is the expected symmetry because of the relation (3.25). The $N \leftrightarrow k$ symmetry corresponds to $\lambda \leftrightarrow (1-\lambda)$ symmetry in the large N limit.

Let us describe the three-point functions.[18] From the diagonal modular invariant with paired identical representations on the left (holomorphic) and the right (antiholomorphic) sectors [28], one of the primaries is given by $(f;0)\otimes(f;0)$, which is denoted by $\mathcal{O}_+$, and the other is given by $(0;f)\otimes(0;f)$, which is denoted by $\mathcal{O}_-$. Then, the three-point functions with these two scalars are obtained, and their ratios can be written as

$$
\frac{<\overline{\mathcal{O}}_+\mathcal{O}_+T^{(1)}>}{<\overline{\mathcal{O}}_-\mathcal{O}_-T^{(1)}>} = \left[\frac{\lambda}{1-\lambda}\right], \qquad \frac{<\overline{\mathcal{O}}_+\mathcal{O}_+T^{(2)}>}{<\overline{\mathcal{O}}_-\mathcal{O}_-T^{(2)}>} = \left[\frac{\lambda}{1-\lambda}\right],
$$

[17] For $N=3$, the corresponding three diagonal matrix elements are given by $-\frac{4(k-3)(23k+31)}{3(k+5)^2(23k+19)}$, the 44 element $\frac{(k+7)(46k+11)}{(k+5)^2(23k+19)}$, and the 55 element $\frac{(46k^2+403k+55)}{(k+5)^2(23k+19)}$. The eigenvalue equation for the zero mode of the higher spin-3 current acting on the 'light' state can be summarized as follows, and its large N 't Hooft limit is also given:

$$
W_0^{(3)}|(f;f)> = -\left[\frac{4(k-N)(5N+16+(6N+5)k)}{3(N+k+2)^2(5N+4+(6N+5)k)}\right]|(f;f)> \to \frac{4\lambda(2\lambda-1)}{3N}|(f;f)> \to 0.
$$

[18] We assume the following normalizations

$$
{}_\pm<(\bar{f};0)|(f;0)>_\pm = {}_\pm<(0;\bar{f})|(0;f)>_\pm, \qquad {}_\pm<(\bar{f};0)|(f;0)>_\mp = {}_\pm<(0;\bar{f})|(0;f)>_\mp = 0.
$$

$$\frac{<\overline{\mathcal{O}}_+\mathcal{O}_+\tilde{W}^{(2)}>}{<\overline{\mathcal{O}}_-\mathcal{O}_-\tilde{W}^{(2)}>} = -\left[\frac{\lambda}{1-\lambda}\right], \qquad \frac{<\overline{\mathcal{O}}_+\mathcal{O}_+W^{(3)}>}{<\overline{\mathcal{O}}_-\mathcal{O}_-W^{(3)}>} = -\left[\frac{\lambda(1+\lambda)}{(1-\lambda)(2-\lambda)}\right]. \quad (3.27)$$

Compared to the bosonic higher spin AdS/CFT duality in the context of the W_N minimal model, the behavior of (3.27) seems similar in the sense that the factor $\frac{\lambda}{(1-\lambda)}$, which is present in the ratios of the three-point function of the higher spin-2 currents appears in the right-hand side of the ratio of the three-point functions of the higher spin-3 current. Furthermore, the factor $\frac{(1+\lambda)}{(2-\lambda)}$ contributes to the final ratio of the three-point functions of the higher spin-3 current. Then, we expect that the ratio of the three-point functions for the higher spin-4 current can be described as $\left[\frac{\lambda(1+\lambda)(2+\lambda)}{(1-\lambda)(2-\lambda)(3-\lambda)}\right]$. Even for the higher spin-s current, we can expect that the ratio of the three-point functions for the higher spin current of spin-s can be written as $\prod_{n=1}^{s-1}\frac{(n-1+\lambda)}{(n-\lambda)}$ up to the sign. Note that the bosonic case has same formula, except the numerator of this expression contains n rather than $(n-1)$ [29]. It would be interesting to study this general spin behavior in detail.

As in footnote 10, we can calculate the sum of the square of the triplet (corresponding to the higher spin-2 currents) in each $\hat{su}(2)$ algebra. For similar calculations in the nonlinear version, where the equation (4.23) of [30] is used, we obtain

$$\begin{aligned}
\left[\sum_{i=1}^{3}\tilde{V}_1^{(1)+i}\tilde{V}_1^{(1)+i}\right]_0 |(f;0)> &= -\left[\frac{12N(5N+4k+2)}{(N+k+2)^2}\right]|(f;0)>,\\
\left[\sum_{i=1}^{3}\tilde{V}_1^{(1)+i}\tilde{V}_1^{(1)+i}\right]_0 |(0;f)> &= -\left[\frac{24k}{(N+k+2)^2}\right]|(0;f)>,\\
\left[\sum_{i=1}^{3}\tilde{V}_1^{(1)-i}\tilde{V}_1^{(1)-i}\right]_0 |(f;0)> &= -\left[\frac{24N}{(N+k+2)^2}\right]|(f;0)>,\\
\left[\sum_{i=1}^{3}\tilde{V}_1^{(1)-i}\tilde{V}_1^{(1)-i}\right]_0 |(0;f)> &= -\left[\frac{12k(5k+4N+2)}{(N+k+2)^2}\right]|(0;f)>. \qquad (3.28)
\end{aligned}$$

Symmetries exist in $N \leftrightarrow k$ and $0 \leftrightarrow f$. As the large N 't Hooft limits are taken in (3.28), they become $-12\lambda(4+\lambda)$, $-\frac{24\lambda(1-\lambda)}{N}$, $-\frac{24\lambda^2}{N}$ and $-12(1-\lambda)(5-\lambda)$.

Therefore, in this section, the ratios of the three-point functions can be summarized by (3.27). The three-point functions can be obtained from the previous relations (3.14), (3.23), and (3.26).

4 The large $\mathcal{N}=4$ linear superconformal algebra and its extension in the coset theory

For the 16 currents of large $\mathcal{N}=4$ linear superconformal algebra, the work of [11] leads to complete expressions in terms of the coset fields. In this section, some of the recapitulation of [11] is given in our notation, and we would like to construct the 16 higher spin currents.

4.1 The 16 currents of $\mathcal{N}=4$ linear superconformal algebra using the Kac-Moody currents

The 16 currents and the 16 higher spin currents are constructed in the following coset theory:

$$\frac{G}{H}=\frac{\mathrm{SU}(N+2)}{\mathrm{SU}(N)}. \tag{4.1}$$

The following three indices are defined in the corresponding group G, subgroup H, and the coset $\frac{G}{H}$, respectively:

$$G \quad \text{indices}: a, b, c, \cdots, \quad H \quad \text{indices}: a', b', c', \cdots, \quad \frac{G}{H} \quad \text{indices}: \tilde{a}, \tilde{b}, \tilde{c}, \cdots. \tag{4.2}$$

The number of coset indices is given by the difference between $(N+2)^2-1$ and (N^2-1), and therefore, the dimension of the coset is given by $(4N+4)$. For a given $(N+2)\times(N+2)$ matrix, the $(4N+4)$ coset indices can be associated with the following locations with asterisks:

$$\left(\begin{array}{ccccc|cc} & & & & & * & * \\ & & & & & * & * \\ & & & & & \vdots & \vdots \\ & & & & & * & * \\ & & & & & * & * \\ \hline * & * & \cdots & * & * & * & * \\ * & * & \cdots & * & * & * & * \end{array}\right)_{(N+2)\times(N+2)} . \tag{4.3}$$

Compared to the previous case in (2.4), there is an extra 2×2 matrix corresponding to $su(2)\oplus u(1)$. We can further divide the linear coset indices (4.2) as $\tilde{a}=(\bar{a},\hat{a})$, where the index $\hat{a}$ is associated with the above 2×2 matrix and runs over 4 values. Of course, the remaining $\bar{a}$ index runs over $4N$ values as in an extension of the large $\mathcal{N}=4$ nonlinear superconformal algebra in section 2.

Let us consider four spin-$\frac{3}{2}$ currents of the large $\mathcal{N}=4$ linear superconformal algebra. It is known that the spin-$\frac{3}{2}$ current corresponding to the $\mathcal{N}=1$ supersymmetry generator consists of two parts. One part contains the spin-1 current as well as the spin-$\frac{1}{2}$ current, while the other part contains the cubic term in the spin-$\frac{1}{2}$ current. (See also [31].) By generalizing the two coefficient tensors to possess the three additional supersymmetry indices, we write

$$\mathbf{G}^{\mu}(z)=A(k,N)\left[h^{\mu}_{\tilde{a}\tilde{b}}\,Q^{\tilde{a}}\,V^{\tilde{b}}+B(k,N)S^{\mu}_{\tilde{a}\tilde{b}\tilde{c}}Q^{\tilde{a}}Q^{\tilde{b}}Q^{\tilde{c}}\right](z), \qquad (\mu=0,1,2,3), \tag{4.4}$$

where the relative coefficients $A(k,N)\equiv\frac{i}{(k+N+2)}$ and $B(k,N)\equiv-\frac{1}{6(k+N+2)}$ are taken from the $\mathcal{N}=1$ supersymmetry generator. Moreover, the two tensors are given by $h^{0}_{\tilde{a}\tilde{b}}\equiv g_{\tilde{a}\tilde{b}}$ and $S^{0}_{\tilde{a}\tilde{b}\tilde{c}}\equiv f_{\tilde{a}\tilde{b}\tilde{c}}$ for the $\mu=0$ index. The new objects $h^{i}_{\tilde{a}\tilde{b}}$ and $S^{i}_{\tilde{a}\tilde{b}\tilde{c}}$ for the other three indices $i=1,2,3$ are undetermined numerical constants.[19]

[19] We use boldface notation for the 16 currents plus the 16 higher spin currents in the linear version. For the previous 11 currents and 16 higher spin currents in the nonlinear version, boldface notation is not used.

JHEP10(2015)111

Then, we obtain the explicit OPE between the spin-$\frac{3}{2}$ current (4.4) and itself as follows:

$$
\begin{aligned}
\mathbf{G}^{\mu}(z)\,\mathbf{G}^{\nu}(w) = & \frac{1}{(z-w)^3}\,A^2\,\Big[-k(k+N+2)\,h^{\mu}_{\tilde{a}\tilde{b}}\,h^{\nu\tilde{a}\tilde{b}}+6B^2(k+N+2)^3 S^{\mu}_{\tilde{a}\tilde{b}\tilde{c}}S^{\nu\tilde{a}\tilde{b}\tilde{c}}\Big] \\
& +\frac{1}{(z-w)^2}\,A^2\,\Big[(k+N+2)\,h^{\mu}_{\tilde{a}\tilde{b}}\,h^{\nu\tilde{a}}{}_{\tilde{d}}\,f^{\tilde{b}\tilde{d}}{}_{e}\,V^{e}+k\,h^{\mu}_{\tilde{a}\tilde{b}}\,h^{\nu\tilde{b}}_{\tilde{c}}\,Q^{\tilde{a}}\,Q^{\tilde{c}} \\
& -18B^2(k+N+2)^2 S^{\mu}_{\tilde{a}\tilde{b}\tilde{c}}S^{\nu\tilde{a}\tilde{b}}{}_{\tilde{d}}\,Q^{\tilde{c}}Q^{\tilde{d}}\Big](w) \\
& +\frac{1}{(z-w)}\,A^2\,\Big[-(k+N+2)\,h^{\mu}_{\tilde{a}\tilde{b}}\,h^{\nu\tilde{a}}{}_{\tilde{d}}\,V^{\tilde{b}}\,V^{\tilde{d}}+k\,h^{\mu}_{\tilde{a}\tilde{b}}\,h^{\nu\tilde{b}}_{\tilde{c}}\,\partial\,Q^{\tilde{a}}\,Q^{\tilde{c}} \\
& -h^{\mu}_{\tilde{a}\tilde{b}}\,h^{\nu}_{\tilde{c}\tilde{d}}f^{\tilde{b}\tilde{d}}{}_{e}\,Q^{\tilde{a}}\,Q^{\tilde{c}}\,V^{e}-6B(k+N+2)S^{(\mu}_{\tilde{a}\tilde{b}\tilde{c}}h^{\nu)\tilde{a}}{}_{\tilde{d}}\,Q^{\tilde{b}}\,Q^{\tilde{c}}\,V^{\tilde{d}} \\
& -9B^2(k+N+2)S^{\mu}_{\tilde{a}\tilde{b}\tilde{c}}S^{\nu\tilde{a}}{}_{\tilde{d}\tilde{e}}\,Q^{\tilde{b}}Q^{\tilde{c}}Q^{\tilde{d}}Q^{\tilde{e}} \\
& -18B^2(k+N+2)^2 S^{\mu}_{\tilde{a}\tilde{b}\tilde{c}}S^{\nu\tilde{a}\tilde{b}}{}_{\tilde{d}}\,\partial Q^{\tilde{c}}Q^{\tilde{d}}\Big](w)+\cdots . \qquad (4.5)
\end{aligned}
$$

Compared to the similar calculation for the OPE between the spin-$\frac{3}{2}$ current and itself in the nonlinear version, the result in (4.5) contains the S^{μ} tensor-dependent terms.

Using the defining $\mathcal{N}=4$ linear superconformal algebra and the above OPE (4.5), we identify the 16 currents in terms of the spin-1 current and spin-$\frac{1}{2}$ current. From the particular expression when $\mu=\nu=0$, we can explicitly determine the spin-2 current. That is, the first-order pole is given by $\mathbf{G}^{0}(z)\,\mathbf{G}^{0}(w)|_{\frac{1}{(z-w)}}=2\mathbf{T}(w)$, and the corresponding expression is obtained from the OPE (4.5). Then, we obtain

$$
\begin{aligned}
\mathbf{T}(z) = & \frac{1}{2(k+N+2)^2}\Big[(k+N+2)\,V_{\tilde{a}}\,V^{\tilde{a}}+k\,Q_{\tilde{a}}\,\partial\,Q^{\tilde{a}}+f_{\tilde{a}\tilde{b}c'}\,Q^{\tilde{a}}\,Q^{\tilde{b}}\,V^{c'} \\
& +\frac{1}{2}f_{\tilde{a}\tilde{b}\tilde{c}}f^{\tilde{a}\tilde{b}}{}_{\tilde{d}}\partial Q^{\tilde{c}}Q^{\tilde{d}}+\frac{1}{4(k+N+2)}f_{\tilde{a}\tilde{b}\tilde{e}}f_{\tilde{c}\tilde{d}}{}^{\tilde{e}}Q^{\tilde{a}}Q^{\tilde{b}}Q^{\tilde{c}}Q^{\tilde{d}}\Big](z). \qquad (4.6)
\end{aligned}
$$

Compared to the corresponding spin-2 current in (2.6), the dummy variables are summed over the extra 4 indices corresponding to the lower 2×2 matrix in (4.3).

From the $V^{\tilde{b}}\,V^{\tilde{d}}(w)$ term of $\mathbf{G}^{(\mu}(z)\,\mathbf{G}^{\nu)}(w)|_{\frac{1}{(z-w)}}=2\delta^{\mu\nu}\mathbf{T}(w)$, we have the identity

$$
h^{\mu}_{\tilde{a}\tilde{b}}\,h^{\nu\tilde{a}}{}_{\tilde{d}}+h^{\nu}_{\tilde{a}\tilde{b}}\,h^{\mu\tilde{a}}{}_{\tilde{d}}=2\,\delta^{\mu\nu}\,g_{\tilde{b}\tilde{d}}, \qquad (\mu,\nu=0,1,2,3), \qquad (4.7)
$$

which corresponds to (2.19) of [11]. The left-hand side of (4.7) comes from the above OPE (4.5), and the right-hand side comes from (4.6). Thus, $h^{i}_{\tilde{a}\tilde{b}}$ are almost complex structures.

From the $Q^{\tilde{a}}\,Q^{\tilde{b}}\,V^{\tilde{e}}$ term of $\mathbf{G}^{(\mu}(z)\,\mathbf{G}^{\nu)}(w)|_{\frac{1}{(z-w)}}=2\delta^{\mu\nu}\mathbf{T}(w)$, the following identities are satisfied:

$$
h^{(\mu}_{\tilde{a}\tilde{c}}h^{\nu)}_{\tilde{b}\tilde{d}}f^{\tilde{c}\tilde{d}}{}_{\tilde{e}}=h^{\tilde{d}}{}_{\tilde{e}}{}^{(\mu}S^{\nu)}_{\tilde{d}\tilde{a}\tilde{b}}, \qquad h^{i}_{\tilde{a}\tilde{c}}f^{\tilde{c}}{}_{\tilde{b}e'}=h^{i}_{\tilde{b}\tilde{c}}f^{\tilde{c}}{}_{\tilde{a}e'}, \qquad (4.8)
$$

which correspond to (2.20) and (2.21) of [11], respectively. The second equation of (4.8) can be found from equation (3.11) of [32]. By using the three identities given in (4.7) and (4.8), the complete expression for the $S^{i}_{\tilde{a}\tilde{b}\tilde{c}}$ tensor is given as follows:

$$
S^{i}_{\tilde{a}\tilde{b}\tilde{c}}=h^{i}_{\tilde{a}\tilde{d}}h^{i}_{\tilde{b}\tilde{e}}h^{i}_{\tilde{c}\tilde{f}}f^{\tilde{d}\tilde{e}\tilde{f}}, \qquad (i=1,2,3), \qquad (4.9)
$$

corresponding to (2.27) of [11].

Thus, the spin-$\frac{3}{2}$ currents, together with the three almost complex structures and the structure constants, where the result in (4.9) is substituted, are given by

$$\mathbf{G}^0(z) = \frac{i}{(k+N+2)}\left[Q_{\tilde{a}}\,V^{\tilde{a}} - \frac{1}{6(k+N+2)}f_{\tilde{a}\tilde{b}\tilde{c}}Q^{\tilde{a}}Q^{\tilde{b}}Q^{\tilde{c}}\right](z),$$
$$\mathbf{G}^j(z) = \frac{i}{(k+N+2)}\left[h^j_{\tilde{a}\tilde{b}}\,Q^{\tilde{a}}\,V^{\tilde{b}} - \frac{1}{6(k+N+2)}h^j_{\tilde{a}\tilde{d}}h^j_{\tilde{b}\tilde{e}}h^j_{\tilde{c}\tilde{f}}f^{\tilde{d}\tilde{e}\tilde{f}}Q^{\tilde{a}}Q^{\tilde{b}}Q^{\tilde{c}}\right](z). \quad (4.10)$$

Compared to the corresponding spin-$\frac{3}{2}$ currents in the nonlinear version, the above expressions (4.10) have a cubic term in the spin-$\frac{1}{2}$ current, and the index $\tilde{a}$ contains 4 indices corresponding to the 2×2 matrix as before.

Let us determine the other currents. The six spin-1 currents $\mathbf{A}^{\pm i}(z)$ can be obtained from the second-order pole $\frac{1}{(z-w)^2}$ terms of the OPE (4.5) and the defining OPEs,[20]

$$\mathbf{A}^{+i}(z) = -\frac{1}{4(N+1)}\left[h^i_{\tilde{a}\tilde{b}}f^{\tilde{a}\tilde{b}}{}_c V^c + \frac{1}{(k+N+2)}\left(h^i_{\tilde{c}\tilde{d}} + \frac{1}{2}S^i_{\tilde{a}\tilde{b}\tilde{c}}f^{\tilde{a}\tilde{b}}{}_d\right)Q^{\tilde{c}}Q^{\tilde{d}}\right](z),$$
$$\mathbf{A}^{-i}(z) = -\frac{1}{4(k+N+2)}h^i_{\tilde{a}\tilde{b}}Q^{\tilde{a}}Q^{\tilde{b}}(z), \quad (4.11)$$

corresponding to (2.36) and (2.37) of [11]. From the second-order poles in the OPEs $\mathbf{A}^{\pm i}(z)\,\mathbf{G}^{\mu}(w)$ with (4.10) and (4.11), the four fermionic spin-$\frac{1}{2}$ currents $\mathbf{\Gamma}^{\mu}(z)$ corresponding to (2.43) of [11] can be fixed as follows:

$$\mathbf{\Gamma}^0(z) = -\frac{i}{4(N+1)}h^j_{\tilde{a}\tilde{b}}f^{\tilde{a}\tilde{b}}{}_{\tilde{c}}h^{j\tilde{c}}{}_{\tilde{d}}Q^{\tilde{d}}(z), \qquad \mathbf{\Gamma}^j(z) = -\frac{i}{4(N+1)}h^j_{\tilde{a}\tilde{b}}f^{\tilde{a}\tilde{b}}{}_{\tilde{c}}Q^{\tilde{c}}(z), \quad (4.12)$$

where $j = 1,2,3$, and there is no sum over j in the first equation of (4.12). From the OPE $\mathbf{\Gamma}^{\mu}(z)\,\mathbf{G}^{\nu}(w)$ when the index μ is the same as the index ν, the spin-1 current $\mathbf{U}(z)$ corresponding to (2.44) of [11] can be determined by

$$\mathbf{U}(z) = -\frac{1}{4(N+1)}h^j_{\tilde{a}\tilde{b}}f^{\tilde{a}\tilde{b}}{}_{\tilde{c}}h^{j\tilde{c}}{}_{\tilde{d}}\left[V^{\tilde{d}} - \frac{1}{2(k+N+2)}f^{\tilde{d}}{}_{\tilde{e}\tilde{f}}Q^{\tilde{e}}Q^{\tilde{f}}\right](z), \quad (4.13)$$

where there is no sum over the index j. We can easily see that the first (second) term of (4.13) comes from the OPE between $\mathbf{\Gamma}^{\mu}(z)$ in (4.12) and the spin-1 term (the cubic term in the spin-$\frac{1}{2}$ current) in $\mathbf{G}^{\mu}(w)$ in (4.10).[21]

See the original arXiv version for the defining OPE equations of the 16 currents and for the explicit form of the complex structures.

[20] We have used the following tensor identities:

$$S^{0\tilde{a}\tilde{b}}{}_{\tilde{c}}S^1_{\tilde{a}\tilde{b}\tilde{d}} + S^{2\tilde{a}\tilde{b}}{}_{\tilde{c}}S^3_{\tilde{a}\tilde{b}\tilde{d}} = 4h^1_{\tilde{c}\tilde{d}}, \quad S^{0\tilde{a}\tilde{b}}{}_{\tilde{c}}S^2_{\tilde{a}\tilde{b}\tilde{d}} + S^{3\tilde{a}\tilde{b}}{}_{\tilde{c}}S^1_{\tilde{a}\tilde{b}\tilde{d}} = 4h^2_{\tilde{c}\tilde{d}}, \quad S^{0\tilde{a}\tilde{b}}{}_{\tilde{c}}S^3_{\tilde{a}\tilde{b}\tilde{d}} + S^{1\tilde{a}\tilde{b}}{}_{\tilde{c}}S^2_{\tilde{a}\tilde{b}\tilde{d}} = 4h^3_{\tilde{c}\tilde{d}}.$$

[21] As in footnote 9, we can calculate the corresponding $\mathbf{U}$ charges for the coset fields as follows:

$$i\mathbf{U}(z)\begin{pmatrix} Q^{\bar{A}} \\ Q^{\bar{A}^*}\end{pmatrix}(w) = \mp\frac{1}{(z-w)}\left[\frac{1}{2}\sqrt{\frac{N+2}{N}}\right]\begin{pmatrix} Q^{\bar{A}} \\ Q^{\bar{A}^*}\end{pmatrix}(w) + \cdots,$$
$$i\mathbf{U}(z)\begin{pmatrix} V^{\bar{A}} \\ V^{\bar{A}^*}\end{pmatrix}(w) = \mp\frac{1}{(z-w)}\left[\frac{1}{2}\sqrt{\frac{N+2}{N}}\right]\begin{pmatrix} V^{\bar{A}} \\ V^{\bar{A}^*}\end{pmatrix}(w) + \cdots.$$

These are proportional to the U charges in footnote 9.

4.2 The higher spin currents in the coset (4.1)

Let us consider the higher spin currents in the extension of the large $\mathcal{N}=4$ linear superconformal algebra. It is crucial to find the lowest higher spin-1 current, and it is straightforward to obtain the remaining higher spin currents. Let us denote the 16 higher spin currents as follows:

$$\left(1,\frac{3}{2},\frac{3}{2},2\right):(\mathbf{T^{(1)}},\mathbf{T_+^{(\frac{3}{2})}},\mathbf{T_-^{(\frac{3}{2})}},\mathbf{T^{(2)}}),\ \left(\frac{3}{2},2,2,\frac{5}{2}\right):(\mathbf{U^{(\frac{3}{2})}},\mathbf{U_+^{(2)}},\mathbf{U_-^{(2)}},\mathbf{U^{(\frac{5}{2})}}),$$
$$\left(\frac{3}{2},2,2,\frac{5}{2}\right):(\mathbf{V^{(\frac{3}{2})}},\mathbf{V_+^{(2)}},\mathbf{V_-^{(2)}},\mathbf{V^{(\frac{5}{2})}}),\ \left(2,\frac{5}{2},\frac{5}{2},3\right):(\mathbf{W^{(2)}},\mathbf{W_+^{(\frac{5}{2})}},\mathbf{W_-^{(\frac{5}{2})}},\mathbf{W^{(3)}}). \quad (4.14)$$

Here, the 16 higher spin currents are primary under the stress-energy tensor given in (4.6). In the convention of [30], the higher spin-3 current is a quasiprimary current under the stress-energy tensor.

4.2.1 The higher spin-1 current

The natural ansatz for the higher spin-1 current is

$$\mathbf{T^{(1)}}(z)=A_a\,V^a(z)+B_{\tilde{a}\tilde{b}}\,Q^{\tilde{a}}\,Q^{\tilde{b}}(z). \quad (4.15)$$

By requiring that the OPEs between the above higher spin-1 current and the six spin-1 currents are regular, the higher spin-1 current is the primary current under the stress-energy tensor $\mathbf{T}(w)$. The OPEs between the higher spin-1 current and the 4 free fermions $\mathbf{\Gamma}^\mu(w)$ (and $\mathbf{U}(w)$) are regular [30], and all unknown coefficients in (4.15) are determined completely. Then, the higher spin-1 current (4.15) in the linear version is the same as the higher spin-1 current in the nonlinear version[22]

$$\mathbf{T^{(1)}}(z)=T^{(1)}(z). \quad (4.16)$$

Now, the lowest higher spin-1 current is completely fixed, and it is straightforward to calculate the remaining higher spin currents as mentioned before.

4.2.2 The higher spin-$\frac{3}{2}$ currents in (4.14)

Let us define the four higher spin-$\frac{3}{2}$ currents $\mathbf{G}'^\mu(w)$ from the first-order pole of the following OPE:

$$\mathbf{G}^\mu(z)\,\mathbf{T^{(1)}}(w)=\frac{1}{(z-w)}\,\mathbf{G}'^\mu(w)+\cdots. \quad (4.17)$$

The first-order pole in (4.17) provides

$$\mathbf{G}'^\mu(z)=G'^\mu(z), \quad (4.18)$$

which is given by (2.13).

[22]Of course, we required the same normalization with the higher spin-1 current $T^{(1)}$ in the nonlinear version and take the same sign as follows:

$$\mathbf{T^{(1)}}(z)\,\mathbf{T^{(1)}}(w)=\frac{1}{(z-w)^2}\left[\frac{2Nk}{N+k+2}\right]+\cdots.$$

By calculating the OPEs between the spin-$\frac{3}{2}$ currents in (4.10) and the higher spin-1 current in (4.16) and by subtracting the spin-$\frac{3}{2}$ currents in the left-hand side with correct coefficients, the following higher spin-$\frac{3}{2}$ currents can be obtained:

$$\begin{aligned}
\mathbf{T}_{+}^{(\frac{3}{2})}(z) &= \frac{1}{2}\left(\mathbf{G}'_{21}-\mathbf{G}_{21}\right)(z) = \frac{1}{2}\left(G'_{21}-\mathbf{G}_{21}\right)(z), \\
\mathbf{T}_{-}^{(\frac{3}{2})}(z) &= \frac{1}{2}\left(\mathbf{G}'_{12}+\mathbf{G}_{12}\right)(z) = \frac{1}{2}\left(G'_{12}+\mathbf{G}_{12}\right)(z), \\
\mathbf{U}^{(\frac{3}{2})}(z) &= \frac{1}{2}\left(\mathbf{G}'_{11}-\mathbf{G}_{11}\right)(z) = \frac{1}{2}\left(G'_{11}-\mathbf{G}_{11}\right)(z), \\
\mathbf{V}^{(\frac{3}{2})}(z) &= \frac{1}{2}\left(\mathbf{G}'_{22}+\mathbf{G}_{22}\right)(z) = \frac{1}{2}\left(G'_{22}+\mathbf{G}_{22}\right)(z).
\end{aligned} \tag{4.19}$$

In the last line of each equation, the conditions in (4.18) are used. Furthermore, the relations in the footnote 8 between the double index and the single index hold in this case. Compared to the corresponding higher spin-$\frac{3}{2}$ currents in the nonlinear version [2], the four corresponding OPEs generating (4.19) are the same as those in the nonlinear version. In other words, those four equations remain unchanged if we use the currents without boldface.

Then, it is straightforward to calculate the 11 remaining higher spin currents of (4.14). See [2, 33] for a detailed description.

5 Three-point functions in the extension of the large $\mathcal{N}=4$ linear superconformal algebra

As in section 3, we calculate the three-point functions for the higher spin currents in the extension of the large $\mathcal{N}=4$ linear superconformal algebra.

5.1 Eigenvalue equations for the spin-2 current acting on the states $|(f;0)>$ and $|(0;f)>$ in the large $\mathcal{N}=4$ linear superconformal algebra

Let us define the $\mathbf{U}$-charge as in [6]:

$$i\mathbf{U}_0|(f;0)>=\mathbf{u}(f;0)|(f;0)>, \qquad i\mathbf{U}_0|(0;f)>=\mathbf{u}(0;f)|(0;f)>.$$

From the explicit expression in (4.13), we can obtain the eigenvalues $\mathbf{u}(f;0)$ and $\mathbf{u}(0;f)$ as follows:

$$\mathbf{u}(f;0) = -\frac{1}{2}\sqrt{\frac{N}{N+2}}, \qquad \mathbf{u}(0;f) = \frac{1}{2}\sqrt{\frac{N+2}{N}}. \tag{5.1}$$

We find the explicit expressions.[23]

[23] For $N=3$, we consider the following matrix acting on the states:

$$i\mathbf{U}_0|(f;\star)>= \left(\begin{array}{ccc|cc}
\frac{1}{\sqrt{15}} & 0 & 0 & 0 & 0 \\
0 & \frac{1}{\sqrt{15}} & 0 & 0 & 0 \\
0 & 0 & \frac{1}{\sqrt{15}} & 0 & 0 \\
\hline
0 & 0 & 0 & -\frac{1}{2}\sqrt{\frac{3}{5}} & 0 \\
0 & 0 & 0 & 0 & -\frac{1}{2}\sqrt{\frac{3}{5}}
\end{array}\right) |(f;\star)>.$$

From the spin-2 current [34], we have the following relation

$$\mathbf{T}_0|(f;0)> \sim \left[T - \frac{1}{(k+N+2)}\mathbf{U}\mathbf{U}\right]_0 |(f;0)> = \left[h(f;0) + \frac{1}{(k+N+2)}\mathbf{u}^2(f;0)\right] |(f;0)>$$
$$= \left[\frac{(N+1)(N+3)}{2(N+2)(k+N+2)}\right] |(f;0)> . \quad (5.2)$$

In the first line, the spin-$\frac{1}{2}$ current-dependent terms are ignored. The eigenvalue in the equation (5.2) is the same value as $h'(f;0)$ in [1]. We can see this by writing $\frac{(N+1)(N+3)}{2(N+2)}$ as $\left[\frac{(N+2)}{2} - \frac{1}{2(N+2)}\right]$, which is the quadratic Casimir of $su(N+2)$ in the fundamental representation. Therefore, the $\hat{u}$ part and u part of [1] in the conformal dimension cancel each other completely.

Because the OPE between $\mathbf{\Gamma}^\mu(z)$ and $Q^{\bar{A}^*}(w)$ is regular, the $\partial \mathbf{\Gamma}^\mu \mathbf{\Gamma}_\mu(z)$ term does not contribute to the eigenvalue equation. Then, we obtain the zero-mode eigenvalue equation of $\mathbf{T}(z)$ for state $|(0;f)>$ as follows:[24]

$$\mathbf{T}_0|(0;f)> \sim \left[T - \frac{1}{(k+N+2)}\mathbf{U}\mathbf{U}\right]_0 |(0;f)> = \left[h(0;f) + \frac{1}{(k+N+2)}\mathbf{u}^2(0;f)\right] |(0;f)>$$
$$= \left[\frac{(Nk+2N+1)}{2N(k+N+2)}\right] |(0;f)> . \quad (5.3)$$

The eigenvalue in the equation (5.3) is exactly the same as $h'(0;f)$ described in [1]. We can also understand this by writing the eigenvalue as $\frac{1}{2} - \left[\frac{(\frac{N}{2} - \frac{1}{2N})}{(k+N+2)}\right]$ where the second term is a quadratic Casimir of $su(N)$ in the fundamental representation and the first term $\frac{1}{2}$ is the conformal dimension from the excitation number. The $\hat{u}$ part and u part of [1] in the conformal dimension cancel each other completely.

The large N limit (3.1) for (5.2) and (5.3) leads to

$$\mathbf{T}_0|(f;0)> = \frac{1}{2}\lambda|(f;0)>, \qquad \mathbf{T}_0|(0;f)> = \frac{1}{2}(1-\lambda)|(0;f)> . \quad (5.4)$$

which are exactly the same as those in subsection 3.1 in the nonlinear version. In other words, the extra $\mathbf{U}\mathbf{U}$ term in (5.2) and (5.3) does not contribute to the eigenvalue equation in this large N limit.

Then, we can vary N for $N = 5, 7, 9$ as before, and we see the general N behavior as in (5.1). We can show that the spin-1 current $\mathbf{U}(z)$ is equivalent to the previous spin-1 current $U(z)$ in footnote 9. In other words, $\mathbf{U}(z) = -\frac{i}{2\sqrt{N(N+2)}}U(z)$. From the explicit form for the spin-1 current, the relevant term can be described as $i\mathbf{U}(z) \sim \frac{1}{2(5+k)}\sqrt{\frac{5}{3}}\sum_{a=1}^{6} Q^a Q^{a^*}(z)$. Then, we can calculate the following OPE: $i\mathbf{U}(z)\, Q^{\bar{A}^*}(w) = \frac{1}{(z-w)}\left[\frac{1}{2}\sqrt{\frac{5}{3}}\right] Q^{\bar{A}^*}(w) + \cdots$. Therefore, the eigenvalue is given by $\frac{1}{2}\sqrt{\frac{5}{3}}$ for $N=3$.

[24] We determine the zero-mode eigenvalue for the 'light' state as follows

$$\mathbf{T}_0|(f;f)> \sim \left[T - \frac{1}{(k+N+2)}\mathbf{U}\mathbf{U}\right]_0 |(f;f)> = \left[h(f;f) + \frac{1}{(k+N+2)}\mathbf{u}^2(f;f)\right] |(f;f)>$$
$$= \left[\frac{(N+1)^2}{N(N+2)(k+N+2)}\right] |(f;f)> \rightarrow \frac{\lambda}{N}|(f;f)> \rightarrow 0,$$

where we used $\mathbf{u}(f;f) = \frac{1}{\sqrt{N(N+2)}}$. See also footnote 23 for the 3×3 diagonal elements, $\frac{1}{\sqrt{15}}$, when $N=3$. In the last line, the large N limit (3.1) is taken.

5.2 Eigenvalue equations for the higher spin-1 current acting on the states $|(f;0)>$ and $|(0;f)>$

In this case, the previous relations (3.12) and (3.13) hold because of (4.16).

5.3 Eigenvalue equations for the higher spin currents of spins 2 and 3 acting on states $|(f;0)>$ and $|(0;f)>$

Let us calculate the eigenvalue equation for the higher spin-2 current. We obtain

$$
\begin{aligned}
\mathbf{T}_0^{(2)}|(f;0)>_{\pm} &= \pm\left[\frac{N}{2(N+k+2)}\right]|(f;0)>_{\pm}, \\
\mathbf{T}_0^{(2)}|(0;f)>_{\pm} &= \pm\left[\frac{k}{2(N+k+2)}\right]|(0;f)>_{\pm}\,.
\end{aligned}
\tag{5.5}
$$

Compared to the previous expression (3.17) in the nonlinear version, the above eigenvalues (5.5) appear in the factors in (3.17). The extra terms in the higher spin-2 current in the linear version also contribute to the eigenvalue equation and can be added to the right-hand side of (3.17). Then, the above simple result occurs. Furthermore, the previous relation (3.18) holds in this case.

For the other higher spin-2 current, a similar calculation gives the following result:

$$
\begin{aligned}
\mathbf{W}_0^{(2)}|(f;0)>_{\pm} &= \mp\left[\frac{N}{2(N+k+2)}\right]|(f;0)>_{\pm}, \\
\mathbf{W}_0^{(2)}|(0;f)>_{\pm} &= \pm\left[\frac{k}{2(N+k+2)}\right]|(0;f)>_{\pm}\,.
\end{aligned}
\tag{5.6}
$$

The eigenvalue equations (5.6) look similar to the previous ones in (5.5), up to the signs. Furthermore, compared to the previous eigenvalue equations in the nonlinear version, we find the eigenvalues in factors of (3.20). In this case, the extra terms in the higher spin-2 current in the linear version also contribute to the eigenvalue equation and can be added to the right-hand side of (3.20). Then, the above very simple result can be obtained. Simple linear combinations between these higher spin-2 currents will also give rise to simple eigenvalue equations, which we show in next subsection. We find the previous relation (3.21) in this case.

Furthermore, if we replace the fundamental representation f with the antifundamental representation $\bar{f}$ in (5.5) and (5.6), the right-hand sides remain unchanged.

By taking the large N 't Hooft limit (3.1), we obtain

$$
\begin{aligned}
\mathbf{T}_0^{(2)}|(f;0)>_{\pm} &= \pm\frac{1}{2}\lambda|(f;0)>_{\pm}, \qquad \mathbf{T}_0^{(2)}|(0;f)>_{\pm}= \pm\frac{1}{2}(1-\lambda)|(0;f)>_{\pm}, \\
\mathbf{W}_0^{(2)}|(f;0)>_{\pm} &= \mp\frac{1}{2}\lambda|(f;0)>_{\pm}, \qquad \mathbf{W}_0^{(2)}|(0;f)>_{\pm}= \pm\frac{1}{2}(1-\lambda)|(0;f)>_{\pm}\,.
\end{aligned}
\tag{5.7}
$$

Surprisingly, these eigenvalue equations (5.7) are exactly the same as those in the nonlinear version (3.23).

For the final higher spin-3 current,[25] the following eigenvalue equations hold:

$$\begin{aligned}
\mathbf{W}_0^{(3)}|(f;0)> &= \mathbf{w^{(3)}}(f;0)|(f;0)>, \qquad \mathbf{W}_0^{(3)}|(0;f)>= \mathbf{w^{(3)}}(0;f)|(0;f)>, \\
\mathbf{w^{(3)}}(f;0) &\equiv \frac{2N}{3(N+2)(N+k+2)^2(4N+5+(3N+4)k)} \times \left[(5N^3+26N^2+50N+30)\right. \\
&\quad \left. + (6N^3+36N^2+71N+43)k + (3N^2+10N+8)k^2\right], \qquad (5.8) \\
\mathbf{w^{(3)}}(0;f) &\equiv -\frac{2k\left[(4N^3+23N^2+18N)+(3N^3+24N^2+16N-3)k+(6N^2+5N)k^2\right]}{3N(N+k+2)^2(4N+5+(3N+4)k)}.
\end{aligned}$$

Based on the partial result in footnote 25, we can further calculate those quantities for $N = 5, 7, 9$. Moreover, the denominators in elements 44 and 55 are not difficult to obtain for generic N. It is nontrivial to see the generic N behavior for the numerators. The numerator is quadratic in k, and we can introduce three independent polynomials in N with the highest power 3 (with four unknown coefficients) in quadratic terms in k, in linear terms in k, and in constant terms. Now, we can use the four constraint equations from the above $N = 3, 5, 7, 9$ cases. The above unknown coefficients are completely and uniquely determined. For the eigenvalue $\mathbf{w^{(3)}}(0;f)$, we can similarly analyze the behavior. The numerator behaves nontrivially. Thus, we introduce three polynomials of order 3 with undetermined coefficients. They can be fixed by solving the relevant equations for $N = 3, 5, 7, 9$.

Furthermore, if we replace the fundamental representation f with the antifundamental representation $\bar{f}$ in (5.8), the right-hand sides have minus signs.

It is obvious that there is no $N \leftrightarrow k$ symmetry, which is different from the $W_0^{(3)}$ case in the nonlinear version. However, under the large N 't Hooft limit (3.1), the above expressions (5.8) have the following simple form:

$$\mathbf{W}_0^{(3)}|(f;0)>= \frac{2}{3}\lambda(1+\lambda)|(f;0)>, \mathbf{W}_0^{(3)}|(0;f)>= -\frac{2}{3}(1-\lambda)(2-\lambda)|(0;f)>. \qquad (5.9)$$

These equations are exactly the same as those in (3.26) for the $W_0^{(3)}$ eigenvalue equations in the nonlinear version.[26]

5.4 Eigenvalue equations for the higher spin currents of spins 2 and 3 acting on states $|(f;0)>$ and $|(0;f)>$ in the basis of [30]

The eigenvalue equations in the nonlinear version [30] are the same as those in the linear version. From the explicit relations in [33],

$$V_1^{(1)\pm 1}(z) = 2i\left(\mathbf{U}_{\mp}^{(2)} - \mathbf{V}_{\pm}^{(2)}\right)(z), \qquad V_1^{(1)\pm 2}(z) = -2\left(\mathbf{U}_{\mp}^{(2)} + \mathbf{V}_{\pm}^{(2)}\right)(z),$$

[25] Explicitly, we obtain the first three diagonal matrix elements $-\frac{52(k-3)(5k+9)}{15(k+5)^2(13k+17)}$, and the remaining two diagonal matrix elements $\frac{2(65k^2+742k+549)}{5(k+5)^2(13k+17)}$ for $N=3$.

[26] We also calculate the eigenvalue equation for the 'light' state as follows:

$$\begin{aligned}
\mathbf{W}_0^{(3)}|(f;f)> &= \left[\frac{4(N-k)(4N^3+19N^2+22N+6+(3N^3+10N^2+8N)k)}{3N(N+2)(N+k+2)^2(4N+5+(3N+4)k)}\right]|(f;f)> \\
&\rightarrow \frac{4\lambda(2\lambda-1)}{3N}|(f;f)> \rightarrow 0.
\end{aligned}$$

As in the footnote 25, the first three elements in the $N=3$ case are relevant to this expression. By following the prescription in (5.8), the explicit N dependence can be fixed completely for $N = 3, 5, 7, 9$. Note that there are two cubic polynomials in N in the numerator.

$$V_1^{(1)\pm 3}(z) = \pm 2i \left(\mathbf{T^{(2)}} \mp \mathbf{W^{(2)}} \right)(z), \tag{5.10}$$

we can rewrite the eigenvalue equations as follows:

$$\begin{aligned} \left[V_1^{(1)+3}\right]_0 |(f;0) >_\pm &= \pm \left[\frac{2iN}{(N+k+2)}\right] |(f;0) >_\pm, \qquad \left[V_1^{(1)+3}\right]_0 |(0;f) >= 0, \\ \left[V_1^{(1)-3}\right]_0 |(f;0) > &= 0, \qquad \left[V_1^{(1)-3}\right]_0 |(0;f) >_\pm = \mp \left[\frac{2ik}{(N+k+2)}\right] |(0;f) >_\pm . \end{aligned} \tag{5.11}$$

The corresponding large N 't Hooft limit (3.1) in (5.11) provides the following result:

$$\begin{aligned} \left[V_1^{(1)+3}\right]_0 |(f;0) >_\pm &= \pm 2i\lambda |(f;0) >_\pm, \qquad \left[V_1^{(1)+3}\right]_0 |(0;f) >= 0, \\ \left[V_1^{(1)-3}\right]_0 |(f;0) > &= 0, \qquad \left[V_1^{(1)-3}\right]_0 |(0;f) >_\pm = \mp 2i(1-\lambda) |(0;f) >_\pm . \end{aligned} \tag{5.12}$$

In this case (5.12), the eigenvalue equation contains zero values, and we cannot find one of the ratios between them.

By introducing the following quantities

$$V_1^{(1)+\pm}(z) \equiv \left[V_1^{(1)+1} \pm i V_1^{(1)+2}\right](z), \qquad V_1^{(1)-\pm}(z) \equiv \left[V_1^{(1)-1} \pm i V_1^{(1)-2}\right](z),$$

we can calculate[27]

$$\begin{aligned} \left[V_1^{(1)+\pm}\right]_0 |(f;0) >_\mp &= - \left[\frac{4Ni}{(N+k+2)}\right] |(f;0) >_\pm \rightarrow -4i\lambda |(f;0) >_\pm, \\ \left[V_1^{(1)-\pm}\right]_0 |(0;f) >_\mp &= \left[\frac{4ki}{(N+k+2)}\right] |(0;f) >_\pm \rightarrow 4i(1-\lambda) |(0;f) >_\pm . \end{aligned}$$

The quadratic combinations will be obtained later.

Furthermore, the following relation [33] which was determined for $N = 3$, together with (4.6) and (4.16) exists:

$$\begin{aligned} V_2^{(1)}(z) &= 4 \left[\mathbf{W^{(3)}} + \frac{4(k-N)}{(4N+5+(3N+4)k)} \left(\mathbf{T}\,\mathbf{T^{(1)}} - \frac{1}{2}\partial^2 \mathbf{T^{(1)}} \right) \right](z) \\ &= 4 \left[\mathbf{W^{(3)}} + \frac{4(k-N)}{(4N+5+(3N+4)k)} \mathbf{T^{(1)}}\,\mathbf{T} \right](z). \end{aligned} \tag{5.13}$$

This holds for any value of N, which is obtained by varying the N values. Note that the N, k dependence in the second term is very simple. In the last line of (5.13), the derivative term is used by changing the commutator [9] in $\left[\mathbf{T^{(1)}}, \mathbf{T}\right](z) = -\frac{1}{2}\partial^2 \mathbf{T^{(1)}}(z)$ in order to simplify the zero-mode eigenvalue equation. For $N = k$, the above spin-3 current $V_2^{(1)}(z)$ becomes a primary current under the stress-energy tensor (4.6).

[27] More precisely, we have

$$\begin{aligned} \left[V_1^{(1)-+}\right]_0 \frac{1}{\sqrt{k+N+2}} Q_{-\frac{1}{2}}^{(a+N)^*} |0 > &= \left[\frac{4ki}{(N+k+2)}\right] \frac{1}{\sqrt{k+N+2}} Q_{-\frac{1}{2}}^{a^*} |0 >, \\ \left[V_1^{(1)--}\right]_0 \frac{1}{\sqrt{k+N+2}} Q_{-\frac{1}{2}}^{a^*} |0 > &= \left[\frac{4ki}{(N+k+2)}\right] \frac{1}{\sqrt{k+N+2}} Q_{-\frac{1}{2}}^{(a+N)^*} |0 >, \end{aligned}$$

where $a = 1, 2, \cdots, N$.

We obtain very simple eigenvalue equations:

$$\left[V_2^{(1)}\right]_0 |(f;0)> \,=\, \left[\frac{8N(2N+k+3)}{3(N+k+2)^2}\right] |(f;0)>,$$
$$\left[V_2^{(1)}\right]_0 |(0;f)> \,=\, -\left[\frac{8k(2k+N+3)}{3(N+k+2)^2}\right] |(0;f)> . \tag{5.14}$$

Note that we can also check the correctness of (5.8) by looking at (5.14) and (5.13) because the zero mode of $\mathbf{T}^{(1)}\mathbf{T}$ acting on the two states is known for general N and k. The coefficient in the second term of (5.13) depends on N, k explicitly. By combining these two contributions, we obtain (5.8) exactly.

From this (5.14), we have the following relation:

$$\left(\left[V_2^{(1)}\right]_0 |(f;0)>\right)_{N\leftrightarrow k, 0\leftrightarrow f} = -\left[V_2^{(1)}\right]_0 |(0;f)> . \tag{5.15}$$

Therefore, in this particular basis, an $N \leftrightarrow k$ symmetry (5.15) exists up to the sign.

The large N 't Hooft limit (3.1) in (5.14) leads to the following result:

$$\left[V_2^{(1)}\right]_0 |(f;0)> \,=\, \frac{8}{3}\lambda(1+\lambda)|(f;0)>,$$
$$\left[V_2^{(1)}\right]_0 |(0;f)> \,=\, -\frac{8}{3}(1-\lambda)(2-\lambda)|(0;f)> . \tag{5.16}$$

Note the four eigenvalues of $\mathbf{W}_0^{(3)}$ are the same as those of $V_2^{(1)}$ in the large N 't Hooft limit (5.16). The second term in the bracket of (5.13) does not contribute to the eigenvalue equation in the large N 't Hooft limit.[28]

We can also calculate the eigenvalue equations for the sum of the square of spin-2 currents acting on the two states with (5.10), and we find

$$\left[\sum_{i=1}^{3} V_1^{(1)+i} V_1^{(1)+i}\right]_0 |(f;0)> \,=\, -\left[\frac{12N(5N+4k+4)}{(N+k+2)^2}\right] |(f;0)>,$$
$$\left[\sum_{i=1}^{3} V_1^{(1)+i} V_1^{(1)+i}\right]_0 |(0;f)> \,=\, -\left[\frac{48k}{(N+k+2)^2}\right] |(0;f)>,$$
$$\left[\sum_{i=1}^{3} V_1^{(1)-i} V_1^{(1)-i}\right]_0 |(f;0)> \,=\, -\left[\frac{48N}{(N+k+2)^2}\right] |(f;0)>,$$
$$\left[\sum_{i=1}^{3} V_1^{(1)-i} V_1^{(1)-i}\right]_0 |(0;f)> \,=\, -\left[\frac{12k(5k+4N+4)}{(N+k+2)^2}\right] |(0;f)> . \tag{5.17}$$

We can interpret that the eigenvalues $-\frac{12N\times 2}{(N+k+2)^2}$, $-\frac{24k}{(N+k+2)^2}$, $-\frac{24N}{(N+k+2)^2}$, and $\frac{-12k\times 2}{(N+k+2)^2}$ appearing in the right-hand side of (5.17) come from the extra terms between the left-hand side

[28] We also sees the following result:

$$\left[V_2^{(1)}\right]_0 |(f;f)> = \left[\frac{16(N-k)}{3(N+k+2)^2}\right] |(f;f)> \rightarrow \frac{16\lambda(2\lambda-1)}{3N}|(f;f)> \rightarrow 0.$$

of (3.28) and the left-hand side of (5.17). An $N \leftrightarrow k$ symmetry exists between the first and last equations (and also the second and third equations) in (5.17). We also have the following large N 't Hooft limits: $-12\lambda(4+\lambda)$, $-\frac{48\lambda(1-\lambda)}{N}$, $-\frac{48\lambda^2}{N}$, and $-12(1-\lambda)(5-\lambda)$. In footnote 10, similar calculations were performed for the spin-1 currents. Here, the conformal dimensions of $V_1^{(1)\pm i}(z)$ are given by 2, and the sum over the $su(2)$ indices in the quadratic of the higher spin-2 currents is taken. It would be interesting to study the representation theory concerning these higher spin currents further by generalizing the previous works in [6]. Under the large N 't Hooft limit, the nonzero eigenvalue corresponding to the quadratic higher spin-2 currents $V_1^{(1)+i}$ appears in the state $|(f;0)>$, while the nonzero eigenvalue corresponding to the quadratic higher spin-2 currents $V_1^{(1)-i}$ appears in the state $|(0;f)>$. The corresponding three-point functions can be described without any difficulty.[29]

Therefore, in this section, the three-point functions can be summarized by (5.4), (5.7), and (5.9). As in the nonlinear version, we obtain the following ratios for the three-point functions:

$$\frac{<\overline{\mathcal{O}}_+\mathcal{O}_+\mathbf{T^{(1)}}>}{<\overline{\mathcal{O}}_-\mathcal{O}_-\mathbf{T^{(1)}}>} = \left[\frac{\lambda}{1-\lambda}\right], \qquad \frac{<\overline{\mathcal{O}}_+\mathcal{O}_+\mathbf{T^{(2)}}>}{<\overline{\mathcal{O}}_-\mathcal{O}_-\mathbf{T^{(2)}}>} = \left[\frac{\lambda}{1-\lambda}\right],$$
$$\frac{<\overline{\mathcal{O}}_+\mathcal{O}_+\mathbf{W^{(2)}}>}{<\overline{\mathcal{O}}_-\mathcal{O}_-\mathbf{W^{(2)}}>} = -\left[\frac{\lambda}{1-\lambda}\right], \qquad \frac{<\overline{\mathcal{O}}_+\mathcal{O}_+\mathbf{W^{(3)}}>}{<\overline{\mathcal{O}}_-\mathcal{O}_-\mathbf{W^{(3)}}>} = -\left[\frac{\lambda(1+\lambda)}{(1-\lambda)(2-\lambda)}\right].$$

These are exactly the same as those in (3.27). Under the large N 't Hooft limit, the ratios of the three-point functions in the nonlinear and linear versions are equivalent.

6 Conclusions and outlook

In this study, the three-point functions in the large $\mathcal{N}=4$ holography are obtained in the large N 't Hooft limit. In the three-point functions, scalar $-$ $\overline{\text{scalar}}$ $-$ current, the two scalars are characterized by the coset primaries corresponding to the two states $|(0;f)>$ and $|(f;0)>$ in the minimal representations of the coset. The currents are given by the spin-1 currents, the spin-2 current of the large $\mathcal{N}=4$ (non)linear superconformal algebra, the higher spin-1 current, the higher spin-2 currents, and the higher spin-3 current.

- Three-point functions in the bulk.

  As described in [29], it is an open problem to obtain the asymptotic symmetry algebra of the higher spin theory on the AdS_3 space. Once this is explicitly determined, then we can compare the results of this study with the corresponding three-point functions, which can be obtained indirectly in the bulk.

- The general s-dependence of the three-point function.

  From the three-point functions with spins 2, 3, we expect that the ratio of the three-point functions for given spin s has the form $\frac{\lambda(1+\lambda)\cdots(s-2+\lambda)}{(1-\lambda)(2-\lambda)\cdots(s-1-\lambda)}$ up to the signs. It

[29] For the light state, we obtain $\left[\sum_{i=1}^{3} V_1^{(1)\pm i} V_1^{(1)\pm i}\right]_0 |(f;f)> = \mathcal{O}(\frac{1}{N})|(f;f)>$, and the corresponding relations in the nonlinear version are given by $\left[\sum_{i=1}^{3} \tilde{V}_1^{(1)\pm i} \tilde{V}_1^{(1)\pm i}\right]_0 |(f;f)> = \mathcal{O}(\frac{1}{N})|(f;f)>$. Therefore, all of these go to zero under the large N 't Hooft limit.

is an open problem to determine whether this is a general behavior. For the higher spin-4 current, we can apply the present method to the orthogonal coset theory, where we have the higher spin-4 current in the lowest $\mathcal{N}=4$ higher spin current. It is a good exercise to see whether we find the above three-point function with $s=4$ and determine whether the behavior looks like that in [35].

- An extension of small $\mathcal{N}=4$ linear superconformal algebra.

  As described in the introduction, the small $\mathcal{N}=4$ linear superconformal algebra can be obtained by taking the large level limit in the large $\mathcal{N}=4$ linear superconformal algebra. Then, we can obtain an extension of the small $\mathcal{N}=4$ linear superconformal algebra from the extension of large $\mathcal{N}=4$ linear superconformal algebra. Therefore, in this construction, the complete OPEs between the 16 currents of large $\mathcal{N}=4$ linear superconformal algebra, and the 16 lowest higher spin currents for general N and k should be obtained.

- Oscillator formalism for the higher spin currents.

  According to the original Vasiliev oscillator formalism, some of the calculations in the higher spin currents of large $\mathcal{N}=4$ nonlinear superconformal algebra are obtained in [1]. It is an open problem to see whether we can see the oscillator formalism in an extension of the large $\mathcal{N}=4$ linear superconformal algebra.

- The operator product expansion of the 16 higher spin currents in $\mathcal{N}=4$ superspace.

  So far, the complete OPEs between the 16 currents and the 16 higher spin currents for general N and k are not known, even though its nonlinear version appears in [30], where the coset field realizations are not verified. The first step is to determine the complete OPEs in an extension of the large $\mathcal{N}=4$ linear superconformal algebra in the $\mathcal{N}=4$ superspace because it is more plausible to consider the linear version rather than the nonlinear version.

- Three-point functions in the coset theory containing an orthogonal Wolf space.

  We can also consider a different large $\mathcal{N}=4$ holography based on the orthogonal Wolf space [36]. The nonlinear version contains the Wolf space $\frac{\mathrm{SO}(N+4)}{\mathrm{SO}(N)\times\mathrm{SU}(2)\times\mathrm{SU}(2)}$, while the linear version contains the coset $\frac{\mathrm{SO}(N+4)}{\mathrm{SO}(N)\times\mathrm{SU}(2)}\times\mathrm{U}(1)$. In this case, the minimal representations contain the two states $|(0;v)>$ and $|(v;0)>$, where the former is the vector representation in $so(N)$ among the singlets in the $so(N+4)$ and the latter is the vector representation in $so(N+4)$ and is simultaneously the singlet under $so(N)$. The relevant previous works in this direction are given in [37–40].

- The next 16 higher spin currents.

  So far, the higher spin currents in the context of the three-point function are members of the 16 lowest higher spin currents. We can consider the next 16 higher spin currents, where the bosonic currents contain the higher spin currents with spins $2,3,4$. We would like to analyze the behaviors of the three-point functions to determine

JHEP10(2015)111

whether they behave as those above. We expect that as the spin increases, the N dependence for several N in the fractional coefficient functions in the level k becomes complicated. In order to extract the general N behavior, we need more information about the OPEs for several N. Furthermore, the basis in [30] is more useful because the defining OPEs between the 16 currents in the large $\mathcal{N} = 4$ linear superconformal algebra and the next 16 higher spin currents have already been presented.

- Three-point functions involving the fermionic (higher spin) currents.

  In [17], the three-point functions, which contain the fermionic (higher spin) currents, have been described. (See also [41].) In this study, we have only considered the bosonic (higher spin) currents in the three-point functions. It would be interesting to explicitly discover the three-point functions with fermionic (higher spin) currents.

Acknowledgments

We would like to thank R. Gopakumar and C. Peng for their helpful discussions. This work was supported by the Mid-career Researcher Program through the National Research Foundation of Korea (NRF) grant funded by the Korean government (MEST) (No. 2012-045385/2013-056327/2014-051185). CA acknowledges warm hospitality from the School of Liberal Arts (and Institute of Convergence Fundamental Studies) at Seoul National University of Science and Technology.

References

[1] M.R. Gaberdiel and R. Gopakumar, *Large-$N = 4$ holography*, *JHEP* **09** (2013) 036 [arXiv:1305.4181] [INSPIRE].

[2] C. Ahn and H. Kim, *Higher spin currents in Wolf space for generic N*, *JHEP* **12** (2014) 109 [arXiv:1411.0356] [INSPIRE].

[3] M.R. Gaberdiel and R. Gopakumar, *An AdS_3 dual for minimal model CFTs*, *Phys. Rev.* **D 83** (2011) 066007 [arXiv:1011.2986] [INSPIRE].

[4] M.R. Gaberdiel and R. Gopakumar, *Triality in minimal model holography*, *JHEP* **07** (2012) 127 [arXiv:1205.2472] [INSPIRE].

[5] M.R. Gaberdiel and R. Gopakumar, *Minimal model holography*, *J. Phys.* **A 46** (2013) 214002 [arXiv:1207.6697] [INSPIRE].

[6] M. Günaydin, J.L. Petersen, A. Taormina and A. Van Proeyen, *On the unitary representations of a class of $N = 4$ superconformal algebras*, *Nucl. Phys.* **B 322** (1989) 402 [INSPIRE].

[7] F.A. Bais, P. Bouwknegt, M. Surridge and K. Schoutens, *Extensions of the Virasoro algebra constructed from Kac-Moody algebras using higher order Casimir invariants*, *Nucl. Phys.* **B 304** (1988) 348 [INSPIRE].

[8] F.A. Bais, P. Bouwknegt, M. Surridge and K. Schoutens, *Coset construction for extended Virasoro algebras*, *Nucl. Phys.* **B 304** (1988) 371 [INSPIRE].

[9] P. Bouwknegt and K. Schoutens, *W symmetry in conformal field theory*, *Phys. Rept.* **223** (1993) 183 [hep-th/9210010] [INSPIRE].

[10] M.R. Gaberdiel and R. Gopakumar, *Higher spins & strings*, *JHEP* **11** (2014) 044 [arXiv:1406.6103] [INSPIRE].

[11] N. Saulina, *Geometric interpretation of the large-$N = 4$ index*, *Nucl. Phys.* **B 706** (2005) 491 [hep-th/0409175] [INSPIRE].

[12] A. Sevrin and G. Theodoridis, *$N = 4$ superconformal coset theories*, *Nucl. Phys.* **B 332** (1990) 380 [INSPIRE].

[13] C. Ahn, *Higher spin currents in Wolf space. Part I*, *JHEP* **03** (2014) 091 [arXiv:1311.6205] [INSPIRE].

[14] C. Ahn, *Higher spin currents in Wolf space. Part II*, *Class. Quant. Grav.* **32** (2015) 015023 [arXiv:1408.0655] [INSPIRE].

[15] K. Thielemans, *A Mathematica package for computing operator product expansions*, *Int. J. Mod. Phys.* **C 2** (1991) 787 [INSPIRE].

[16] T. Creutzig, Y. Hikida and P.B. Ronne, *Extended higher spin holography and Grassmannian models*, *JHEP* **11** (2013) 038 [arXiv:1306.0466] [INSPIRE].

[17] H. Moradi and K. Zoubos, *Three-point functions in $N = 2$ higher-spin holography*, *JHEP* **04** (2013) 018 [arXiv:1211.2239] [INSPIRE].

[18] T. Creutzig and Y. Hikida, *Higgs phenomenon for higher spin fields on AdS_3*, arXiv:1506.04465 [INSPIRE].

[19] Y. Hikida and P.B. Rønne, *Marginal deformations and the Higgs phenomenon in higher spin AdS_3 holography*, *JHEP* **07** (2015) 125 [arXiv:1503.03870] [INSPIRE].

[20] M.R. Gaberdiel and C. Peng, *The symmetry of large $N = 4$ holography*, *JHEP* **05** (2014) 152 [arXiv:1403.2396] [INSPIRE].

[21] V.G. Kac and I.T. Todorov, *Superconformal current algebras and their unitary representations*, *Commun. Math. Phys.* **102** (1985) 337 [INSPIRE].

[22] C.M. Hull and B.J. Spence, *$N = 2$ current algebra and coset models*, *Phys. Lett.* **B 241** (1990) 357 [INSPIRE].

[23] Landau Institute for Theoretical Physics collaboration, S.E. Parkhomenko, *QuasiFrobenius Lie algebras construction of $N = 4$ superconformal field theories*, *Mod. Phys. Lett.* **A 11** (1996) 445 [hep-th/9503071] [INSPIRE].

[24] J.M. Figueroa-O'Farrill, *$N = 2$ structures on solvable Lie algebras: the $c = 9$ classification*, *Commun. Math. Phys.* **177** (1996) 129 [hep-th/9412008] [INSPIRE].

[25] M.R. Gaberdiel and T. Hartman, *Symmetries of holographic minimal models*, *JHEP* **05** (2011) 031 [arXiv:1101.2910] [INSPIRE].

[26] A. Sevrin, W. Troost and A. Van Proeyen, *Superconformal algebras in two-dimensions with $N = 4$*, *Phys. Lett.* **B 208** (1988) 447 [INSPIRE].

[27] M. Ammon, P. Kraus and E. Perlmutter, *Scalar fields and three-point functions in $D = 3$ higher spin gravity*, *JHEP* **07** (2012) 113 [arXiv:1111.3926] [INSPIRE].

JHEP10(2015)111

[28] C.-M. Chang and X. Yin, *Higher spin gravity with matter in AdS_3 and its CFT dual*, JHEP **10** (2012) 024 [arXiv:1106.2580] [INSPIRE].

[29] C. Ahn and H. Kim, *Spin-5 Casimir operator its three-point functions with two scalars*, JHEP **01** (2014) 012 [*Erratum ibid.* **01** (2014) 174] [arXiv:1308.1726] [INSPIRE].

[30] M. Beccaria, C. Candu and M.R. Gaberdiel, *The large-$N = 4$ superconformal W_∞ algebra*, JHEP **06** (2014) 117 [arXiv:1404.1694] [INSPIRE].

[31] L. Álvarez-Gaumé and D.Z. Freedman, *Geometrical structure and ultraviolet finiteness in the supersymmetric σ-model*, Commun. Math. Phys. **80** (1981) 443 [INSPIRE].

[32] S.J. Gates, Jr. and S.V. Ketov, *No $N = 4$ strings on Wolf spaces*, Phys. Rev. **D 52** (1995) 2278 [hep-th/9501140] [INSPIRE].

[33] C. Ahn, *Higher spin currents in Wolf space: III*, Class. Quant. Grav. **32** (2015) 185001 [arXiv:1504.00070] [INSPIRE].

[34] P. Goddard and A. Schwimmer, *Factoring out free fermions and superconformal algebras*, Phys. Lett. **B 214** (1988) 209 [INSPIRE].

[35] C. Ahn, *The coset Spin-4 Casimir operator and its three-point functions with scalars*, JHEP **02** (2012) 027 [arXiv:1111.0091] [INSPIRE].

[36] C. Ahn and J. Paeng, *Higher spin currents in orthogonal Wolf space*, Class. Quant. Grav. **32** (2015) 045011 [arXiv:1410.0080] [INSPIRE].

[37] C. Ahn, *The large-N 't Hooft limit of coset minimal models*, JHEP **10** (2011) 125 [arXiv:1106.0351] [INSPIRE].

[38] M.R. Gaberdiel and C. Vollenweider, *Minimal model holography for* SO($2N$), JHEP **08** (2011) 104 [arXiv:1106.2634] [INSPIRE].

[39] C. Ahn, *The primary Spin-4 Casimir operators in the holographic* SO(N) *coset minimal models*, JHEP **05** (2012) 040 [arXiv:1202.0074] [INSPIRE].

[40] C. Ahn and J. Paeng, *Higher spin currents in the holographic $N = 1$ coset minimal model*, JHEP **01** (2014) 007 [arXiv:1310.6185] [INSPIRE].

[41] T. Creutzig, Y. Hikida and P.B. Ronne, *Three point functions in higher spin AdS_3 supergravity*, JHEP **01** (2013) 171 [arXiv:1211.2237] [INSPIRE].

JHEP10(2015)111